TOPOLOGICAL ALGEBRAIC GEOMETRY
COURSE AT UNIV. OF COPENHAGEN JUNE 16-20, 2008.
10 LECTURES OF ~ 45 MINUTES EACH

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Synopsis: I would like to give a concise introduction to derived algebraic geometry over the moduli stack of formal groups. Put another way, I would like to discuss when continuous families of Landweber exact homology theories can be lifted to families of structured ring spectra. I would also like to explain why we care: this is part of long standing program (going back to Morava and others) of using arithmetic algebraic geometry to understand phenomena in stable homotopy theory. While very much a developing theory, the work of Hopkins, Miller, Lurie, Behrens, Lawson, and others over the last ten or so years have given us very precise tools. The theory and practice of these tools is the emphasis of these lectures. Topics we be a subset of the following list, depending on time and the background of the audience.

1. Schemes, stacks, and algebraic stacks
2. Structured ring spectra and derived algebraic geometry
3. Formal groups, p-divisible groups, and their role in homotopy theory
4. Derived stacks over the moduli stack of formal groups; Lurie's Theorem
5. Deformations of formal groups and the Hopkins-Miller theorem
6. Examples: the Elliptic Case (Topological Modular Forms)
7. Examples: Shimura varieties (Topological Automorphic Forms)

General Prerequisites: I will assume that members of the audience have a background in the basics of stable homotopy theory, including some knowledge of complex oriented cohomology theories, formal groups, their interaction and applications. It would also be helpful if listeners were conversant with the standard concepts of algebraic geometry such as schemes, sheaves of modules, and cohomology, although this I will develop to a certain extent.

Specific things you might want to look at:

We will assume a certain amount of basic stable homotopy theory and algebraic geometry, including:

1. Basic theory of complex orientable homology theories, formal group laws, and the Adams-Novikov spectral sequence as can be found in
– Ravenel, "Complex Cobordism and Stable Homotopy Groups of Spheres", Chaps 2.2, 3.1, 4.1, 4.4, Appendix 2

2. Schemes, their sheaves of modules, and cohomology
– Hartshorne, Chaps 2.2, 2.3; 3.9, 3.10 (Basics on schemes)
– Hartshorne, Chaps 2.5, 3.2, 3.4 (Quasi-coherent sheaves, cohomology)

The rest of the material below will be reviewed, sometimes very quickly, and some prior familiarity could only be helpful.

3. Schemes as functors
– Eisenbud and Harris, "Geometry of Schemes", Chaps I and VI
– Demazure and Gabriel, Chaps 1.1, 1.2, 1.3
– Demazure and Gabriel 2.1, 2.4 (Quasi-coherent sheaves, redux)

4. Basic theory of algebraic stacks and their module sheaves
– Appendix to Vistoli, "Intersection theory on algebraic stacks", Invent. Math. 97 (1989).
– Laumon and Moret-Bailly, "Champs Algébriques", Chaps 4 and 13.

5. Spectra, structured ring spectra, and the stable homotopy category
– Bousfield-Friedlander "Homotopy theory of Gamma-spaces, spectra, and bisimplicial sets" especially the sections about spectra.
– Schwede, "An untitled book project about symmetric spectra", especially Chapter 1.

6. Formal groups, p-divisible groups – Messing, "The Crystals Associated to Barsotti-Tate groups: with Applications to Abelian Schemes", Chaps I and II.1