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18.715: Topics in homological algebra

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The purpose of this course is to give an introduction to basic cyclic theory in general and to the topological cyclic homology functor in particular. Here is a list of topics to be covered in the first couple of months:

- cyclic sets and their geometric realization, edgewise subdivision
- Hochschild homology $\mathrm{HH}(A)$ and the cyclic bar construction
- cyclic homology $\mathrm{HC}(A)$
- symmetric ring spectra
- the topological Hochschild space $\mathrm{THH}(L)$
- topological cyclic homology $\mathrm{TC}(L)$
- basic equivariant stable homotopy theory
- cyclotomic spectra and the fundamental cofibration sequence

$$T_{hC_{p^n}} \xrightarrow{N} T^{C_{p^n}} \xrightarrow{R} T^{C_{p^{n-1}}}$$

- Witt vectors and TC_{-1}
- the norm cofibration sequence

$$T_{hG} \xrightarrow{N^h} T^{hG} \xrightarrow{R^h} \hat{\mathbb{H}}(G;T)$$

and the Tate spectral sequence

- the basic calculation of $\mathrm{TC}(k)$ for k a perfect field of positive characteristic

After the basic material, and if time permits, we can go into more advanced topics in the theory. One possibility is the relation to algebraic K-theory given by the cyclotomic trace. Another direction is the connection with the de Rham-Witt complex and cristalline cohomology. Finally, there are more calculational results, *e.g.* topological cyclic homology of monoid algebras.

Prerequisites: some familiarity with spectra, simplicial sets and spectral sequences.