18.099: Problem Set 2

Due: Tuesday, September 16. (Please slip homework under my office door 2-269 by noon.)

Write proofs of the following statements. The proof of the last two statements uses the least upper bound property of the real numbers. Please be sure that you point this out in your argument. The proof of the last statement also uses the following property of the set $\mathbb{N}$ of positive integers: Every non-empty subset $E \subset \mathbb{N}$ has a smallest element.

1. If both $\alpha, \beta \in \mathbb{R}$ are least upper bounds of $E \subset \mathbb{R}$, then $\alpha = \beta$.

2. If $x, y \in \mathbb{R}$ and if $x > 0$, then there exists $n \in \mathbb{N}$ such that $n \cdot x > y$.

3. For all $x \in \mathbb{R}$, there exists a unique integer $m \in \mathbb{Z}$ such that $m \leq x < m + 1$.