18.099: Problem Set 4

Due: Tuesday, October 7.

1. Let $F \subset \mathbb{R}$ be a closed subset and suppose that $\alpha = \sup F$ exists. Show that $\alpha \in F$. (*Hint:* If $\alpha \notin F$ then $\alpha \in \mathbb{R} \setminus F$.)

2. Let X be a metric space, let $F \subset X$ be a closed subset, and let $K \subset X$ be a compact subset. Show that $F \cap K \subset X$ is a compact subset.

3. Let X be a set. The *discrete metric* on X is given by the map

$$d \colon X \times X \to \mathbb{R}$$

that takes (p, q) to 0 and 1, respectively, as $p = q$ and $p \neq q$. Show that every subset $E \subset X$ is open. Conclude that every subset $F \subset X$ is closed. Finally, which subsets $K \subset X$ are compact?