18.917: *Topics in algebraic topology*

Time and place: TR 1-2:30 in 2-142.

This is a course on algebraic cycles in general and the recent result of Rülling that identifies the additive higher Chow groups defined by Bloch and Esnault as groups of big de Rham-Witt forms in particular. The pace of the course and the amount of back ground material will depend on the attendants.

One expects that, for every scheme X, there exists a spectral sequence

$$E^2_{s,t} = H^{t-s}(X, \mathbb{Z}(t)) \Rightarrow K_{s+t}(X)$$

from the motivic cohomology of X to the algebraic K-theory of X. It is also generally expected that, for X a *regular* scheme, the motivic cohomology groups are given by the higher Chow groups defined by Bloch

$$H^{t-s}(X, \mathbb{Z}(t)) = \mathrm{CH}^t(X, s+t).$$

Indeed, both statements are known to be true, if X is a smooth variety over a field. However, if X is not a regular scheme, then this definition of motivic cohomology is not correct. For instance, the canonical map $X_{\mathrm{red}} \hookrightarrow X$ induces an isomorphism of higher Chow groups but not of algebraic K-groups. In fact, at present, there is no candidate for the definition of motivic cohomology of non-regular schemes. Similarly, there is no general conjecture for the structure of the algebraic K-theory for non-regular schemes.

Bloch and Esnault has proposed a definition of the group $H^t(X, \mathbb{Z}(t))$ in the special case where $X = \operatorname{Spec} k[t]/(t^m)$ is the prime spectrum of the truncated polynomial algebra over a field k. This group naturally decomposes as a direct sum

$$H^t(X, \mathbb{Z}(t)) = H^t(\operatorname{Spec} k, \mathbb{Z}(t)) \oplus \tilde{H}^t(X, \mathbb{Z}(t)).$$

The structure of the algebraic K-theory of X is well-understood by work of Madsen and myself. It is expected that there should be an isomorphism

$$\mathbf{W}_m\Omega^{t-1}_k \xrightarrow{\sim} \tilde{H}^t(X, \mathbb{Z}(t))$$

where the left-hand term is the group of big de Rham-Witt $(t-1)$-forms of length m on k. The theorem of Rülling establishes this isomorphism.

Here is a possible outline of the course.

(1) Schemes and algebraic cycles.
(2) Higher Chow groups and Milnor K-theory.
(3) Additive higher Chow groups.
(4) De Rham-Witt forms and transfers.
(5) Proof of the theorem.