Report problems

Due: Thursday, January 29, 2009, before 5:00 pm at Bldg. Sci. 1, room 105.

Problem 1: Find the point (x_1, x_2, x_3) where the function

$$f(x_1, x_2, x_3) = x_1 + x_2 + x_3$$

takes its maximum value subject to the constraints that x_1, x_2, and x_3 be non-negative and satisfy the linear inequalities

$$\begin{aligned} x_1 + x_2 + x_3 &\leqslant 12 \\ 2x_1 - x_2 - x_3 &\leqslant 9 \\ x_1 + 3x_2 + x_3 &\leqslant 18 \end{aligned}$$

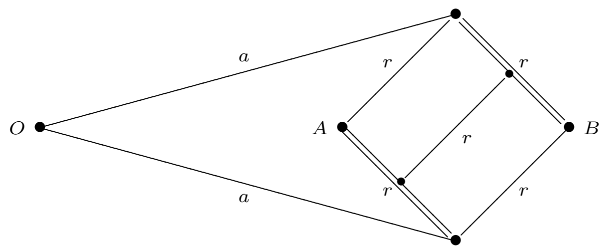
(Answer check: The maximum value is 12.)

Problem 2: Let $(0, 1)$ be the open interval. Does every continuous function

$$f\colon (0,1) \to (0,1)$$

have a fixed point? If yes, give a proof. If no, give a counter-example.

Problem 3: Show that the Peaucellier-Lipkin inversor



inverts. More precisely, define $t > 0$ by $a^2 = r^2 + t^2$. Then, if the vertex O is placed at the origin of $\mathbb{C}$, and if the vertex A is placed at $z \in \mathbb{C}$ with $a - r < |z| < t$, show that the position of the vertex B is $w = t^2/\bar{z} \in \mathbb{C}$.