代数トポロジー特論 II

ヘッセルホルト・ラース

Evaluation: Occational exercises reviewed by the teacher.

Object of the course: We introduce stable homotopy theory and generalized homology theories. The stable homotopy category is a tensor triangulated category and every generalized homology and cohomology theory is of the form

$$E_n(X) = \mathrm{Hom}(S^n, X \wedge E)$$
$$E^n(X) = \mathrm{Hom}(X, S^n \wedge E)$$

for some object E of the stable homotopy category. We construct the stable homotopy category as the homotopy category of symmetric spectra. We discuss several examples of symmetric spectra including the sphere spectrum $\mathbb{S}$ which represents stable homotopy $\pi_*^S(X)$ and stable cohomotopy $\pi_S^*(X)$; the symmetric Eilenberg-MacLane spectrum HA which represents singular homology $H_*(X;A)$ and singular cohomology $H^*(X;A)$; the symmetric spectrum KU which represents topological K-theory $K^*(X)$ and the topological K-homology $K_*(X)$; and the Thom spectrum MU which represents complex cobordism $MU_*(X)$.

Schedule of the course: We first introduce the notion of symmetric spectra and use these to define the stable homotopy category. We show that the category of symmetric spectra admits a Quillen model structure and that the associated homotopy category is the stable homotopy category.

Keywords: Homotopy, model categories, symmetric spectra, algebraic K-theory.

Required knowledge: An introductory course in algebraic topology including the fundamental group and covering spaces.

Text:

Stefan Schwede, *An Untitled Book Project about Symmetric Spectra*, available for free download at `http://www.uni-bonn.de/people/schwede/SymSpec.pdf`.

Friedhelm Waldhausen, *Algebraic K-theory of spaces*, Lecture Notes in Mathematics, vol. 1126, Springer-Verlag, New York.