

## Algebraic Topology: Problem Set 2

Due: Friday, December 18.

In Lecture 9, we constructed, for every map  $f: (X, x) \rightarrow (Y, y)$  of pointed  $k$ -spaces, the following sequence of maps of pointed  $k$ -spaces:

$$(\Omega(Y, y), \bar{y}) \xrightarrow{\partial} (F(f, y), (x, \bar{y})) \xrightarrow{i} (X, x) \xrightarrow{f} (Y, y)$$

We define a sequence of maps of pointed  $k$ -spaces

$$(\Omega(C, c), \bar{c}) \xrightarrow{g} (A, a) \xrightarrow{h} (B, b) \xrightarrow{k} (C, c)$$

to be a (*Hurewicz*) *fiber sequence* if there exists a map  $f: (X, x) \rightarrow (Y, y)$  of pointed  $k$ -spaces and a homotopy commutative diagram of pointed  $k$ -spaces

$$\begin{array}{ccccccc} (\Omega(Y, y), \bar{y}) & \xrightarrow{\partial} & (F(f, y), (x, \bar{y})) & \xrightarrow{i} & (X, x) & \xrightarrow{f} & (Y, y) \\ \downarrow \Omega(\psi) & & \downarrow \eta & & \downarrow \varphi & & \downarrow \psi \\ (\Omega(C, c), \bar{c}) & \xrightarrow{g} & (A, a) & \xrightarrow{h} & (B, b) & \xrightarrow{k} & (C, c) \end{array}$$

such that the vertical maps  $\eta$ ,  $\varphi$ , and  $\psi$  are homotopy equivalences. Here  $\Omega(\psi)$  denotes the map of loop spaces induced by the map  $\psi$ . Prove that, for every map  $f: (X, x) \rightarrow (Y, y)$  of pointed  $k$ -spaces, the sequence of maps of pointed  $k$ -spaces

$$(\Omega(X, x), \bar{x}) \xrightarrow{-\Omega(f)} (\Omega(Y, y), \bar{y}) \xrightarrow{\partial} (F(f, y), (x, \bar{y})) \xrightarrow{i} (X, x),$$

where the map  $-\Omega(f)$  is defined by

$$-\Omega(f)(\omega)(t) = f(\omega(-t)),$$

is a fiber sequence.