2009 年度 前期	対象学年	4 年	レベル	3	2 単位	A 類 II (専門科目)

【科 目 名】幾何学概論 II
ホモトピー論入門

【担当教員】ヘッセルホルト　ラース

【成績評価方法】レポートの結果による判断します。

【教科書および参考書】Course lecture notes will be handed out before each lecture. The following additional texts are also helpful.

[1] Mark Hovey, *Model categories*, Mathematical Surveys and Monographs, vol. 63, American Mathematical Society.

[2] Daniel G. Quillen, *Homotopical algebra*, Lecture Notes in Math., vol. 43, Springer-Verlag.

【講義の目的】To every category $\mathcal{C}$, we associate a topological space $B\mathcal{C}$ called the *classifying space* of $\mathcal{C}$. A functor $f\colon \mathcal{C} \to \mathcal{D}$ gives rise to a continuous map $Bf\colon B\mathcal{C} \to B\mathcal{D}$, and a natural transformation α from the functor $f\colon \mathcal{C} \to \mathcal{D}$ to the functor $g\colon \mathcal{C} \to \mathcal{D}$ gives rise to a homotopy $B\alpha\colon B\mathcal{C} \times [0,1] \to B\mathcal{D}$ from the map Bf to the map Bg. In this way, properties of categories are reflected in the homotopy type of their classifying spaces and vice versa.

【講義予定】The classifying space is constructed by gluing together simplices

$$\Delta[n] = \{(x_0, \dots, x_n) \in [0,1]^{n+1} \mid x_0 + \dots + x_n = 1\}.$$

The general recipe for constructing a topological space by gluing together simplices is called a *simplicial set* and the resulting topological space is called the *geometric realization* of the simplicial set. The first part of the course will focus on simplicial sets and their geometric realization along with the basic category theoretical notions of limits and colimits and adjoints functors which are needed to develop this theory. The next part of the course focuses on homotopy theory. We introduce homotopy groups and define a continuous map between topological spaces to be a *weak equivalence* if it induces an isomorphism of the associated homotopy groups. The *homotopy category* of topological spaces is defined to be the category obtained by formally introducing an inversa of every weak equivalence. The main techniques for studying the homotopy category are centered around two classes of maps called *fibrations* and *cofibrations.* The category of topological spaces together with the three classes of maps given by the weak equivalences, the fibrations, and the cofibrations form a *model category.* In homotopy theory, theorems live in the homotopy category, but the proofs of theorems live in the model category. Finally, we use the homotopy theoretical techniques to define higher algebraic K-theory and, in particular, to prove the so-called additivity theorem.

【キーワード】Homotopy theory, simplicial sets, model categories, higher algebraic K-theory

【履修に必要な知識】学部で学ぶ解析, 幾何, 代数の基礎知識。

【他学科学生の聴講】歓迎します。

【履修の際のアドバイス】基本的なトポロジー、基本群、被覆空間

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