Pespectives in Mathematical Sciences: Report Problems

Due: Tuesday, November 10.

We recall that the abelian group A is said to be generated by the subset $S \subset A$ if every element $a \in A$ can be expressed as a $\mathbb{Z}$-linear combination

$$a = \sum_{s \in S} n_s s$$

where all but finitely many of the integers n_s are zero.

PROBLEM 1. Given positive real numbers a and b, we let $T(a,b) \subset \mathbb{R}^2$ be the right triangle with vertices $(0,0)$, $(a,0)$, and (a,b). Show that the subset

$$S = \{[T(a,b)] \mid a, b \in (0,\infty)\} \subset P(\mathbb{R}^2)$$

generates the scissors congruence group $P(\mathbb{R}^2)$.

PROBLEM 2. Given positive real numbers a, b, and c, we define $T(a,b,c) \subset \mathbb{R}^3$ to be the tetrahedron with vertices $(0,0,0)$, $(a,0,0)$, $(a,b,0)$, and (a,b,c). We will say that $T(a,b,c)$ is a right tetrahedron. Show that the subset

$$S = \{[T(a,b,c)] \mid a, b, c \in (0,\infty)\} \subset P(\mathbb{R}^3)$$

generates the scissors congruence group $P(\mathbb{R}^3)$.