0118

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\begin{tikzcd}
(e_1, e_2) \arrow[rrr, bend left] & & & (e_1, e_2) \\
& \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R}^2 & \\
& \mathbb{R}^2 \arrow[r, "f"] \arrow[u, "id"] & \mathbb{R}^2 \arrow[u, "id"] & \\
(a_1, a_2) \arrow[uuu, bend left, "P"] \arrow[rrr, bend right, "B"] & & & (a_1, a_2) \arrow[uuu, bend right, "P"']
\end{tikzcd}$$

$$B = P^{-1} A P$$

$$= -\frac{1}{3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$$

$$P = \left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]\begin{array}{l} \\ +R_1 \\ +(-1)R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{array}\right]\begin{array}{l} \\ +R_2 \\ \times(-1) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \end{array}\right]\begin{array}{l} +(-1)R_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$= P^{-1}$$

$$
\begin{tikzcd}
(e_1, e_2, e_3) \arrow[rrr, bend left, "A"] & & & (e_1, e_2, e_3) \\
& \mathbb{R}^3 \arrow[r, "f"] & \mathbb{R}^3 & \\
& \mathbb{R}^3 \arrow[r, "f"] \arrow[u, "id"'] & \mathbb{R}^3 \arrow[u, "id"'] & \\
(b_1, b_2, b_3) \arrow[uuu, "P"] \arrow[rrr, bend right, "B"'] & & & (b_1, b_2, b_3) \arrow[uuu, "P"']
\end{tikzcd}
$$

$$
\begin{aligned}
A &= P B P^{-1} \\
&= \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & -1 \\ 0 & 2 & 2 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 1 & -2 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}
\end{aligned}
$$

CH27

(a) $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix} + (-1) R_1$

$+$

(c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 3 \end{bmatrix}$ $+ (-2) R_2$ $\times (-1)$ $\xrightarrow[\text{then}]{}$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$

Basis for ker(f) is :

$b_1 \cdot (-2) + b_2 + b_3$.

(d) $U \xrightarrow{\mathrm{id}_U} U$

$P = \begin{bmatrix} 2 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$(a_1', a_2', a_3') \longrightarrow (a_1, a_2, a_3)$

$$\left[\begin{array}{ccc|ccc} 2 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & -1 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & 1 & 1 & -1 & -2 \end{array}\right]$$

$$U \xleftarrow{\mathrm{id}} U$$

$$\begin{bmatrix} 1 & -2 & -1 \\ -1 & 2 & 2 \\ 1 & -1 & -2 \end{bmatrix}$$

$$(a_1', a_2', a_3') \leftarrow (a_1, a_2, a_3)$$

$$V \xrightarrow{\mathrm{id}_V} V$$

$$Q = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$(b_1', b_2') \rightarrow (b_1, b_2)$$

$$V \xleftarrow{\mathrm{id}_V} V$$

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$(b_1', b_2') \longleftarrow (b_1, b_2)$$

(e) $\begin{bmatrix} 1 & -2 & -1 \\ -1 & 2 & 2 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$

$$\therefore \quad u = a_1' \cdot 3 - a_2' \cdot 3 + a_3' \cdot 2$$

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$\therefore \quad v = -b_1' \cdot 3 + b_2' \cdot 2 .$$

(f)

$$\begin{tikzcd}
(a_1, a_2, a_3) \arrow[rrr, "A"] & & & (b_1, b_2) \\
& U \arrow[r, "f"] & V & \\
& U \arrow[r, "f"] \arrow[u, "id_U"'] & V \arrow[u, "id_V"'] & \\
(a_1', a_2', a_3') \arrow[rrr, "B"] \arrow[uuu, "P"'] & & & (b_1', b_2') \arrow[uuu, "Q"']
\end{tikzcd}$$

$$B = Q^{-1} A P$$

$$= \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -4 & 6 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & -2 \\ -3 & 0 & 3 \end{bmatrix}$$

(g) $$\begin{bmatrix} 8 & 5 & -2 \\ -3 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\therefore f(u) = b_1' \cdot 5 - b_2' \cdot 3$$

(h) $$V \xrightarrow{g} U$$

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$(b_1, b_2) \longrightarrow (a_1, a_2, a_3)$$

H30

(a)

$$\begin{tikzcd}
\mathbb{R}^3 \arrow[r, "f"] & \mathbb{R}^3 \\
(a,b,c) \arrow[r, "{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = B}"] & (a,b,c)
\end{tikzcd}$$

(c)

$$\begin{tikzcd}
(e_1,e_2,e_3) \arrow[rrr, "A"] & & & (e_1,e_2,e_3) \\
& \mathbb{R}^3 \arrow[r, "f"] & \mathbb{R}^3 & \\
& \mathbb{R}^3 \arrow[r, "f"] \arrow[u, "id"'] & \mathbb{R}^3 \arrow[u, "id"'] & \\
(a,b,c) \arrow[rrr, "B"] \arrow[uuu, "{P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}"] & & & (a,b,c) \arrow[uuu, "{P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}"']
\end{tikzcd}$$

$$A = PBP^{-1}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$