18.727: Problem Set 4

Due: 3/14/01

1. Let μ_n be the affine group scheme whose R-valued points are

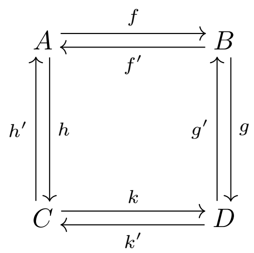
$$\mu_n(R) = \{x \in R \,|\, x^n = 1\}$$

with the groups structure given by multiplication. Describe the Hopf algebra which represents μ_n.

2. Show that separated morphisms are preserved under base change. This means that if $f\colon X \to S$ is a separated morphism and if $g\colon S' \to S$ is any morphism, then the projection $f' = p_2\colon X \times_S S' \to S'$ is a separated morphism. (*Hint*: use the definition.)

3. Let $f\colon X \to S$ be a morphism. Show that f is separated (resp. proper) if and only if S has an open covering $\{U_i\}_{i \in I}$ such that for all $i \in I$, the induced map $f^{-1}(U_i) \to U_i$ is separated (resp. proper).

4. Consider the categories and functors



and assume that f (resp. g, resp. h, resp. k) is left adjoint to f' (resp. g', resp. h', resp. k'). Show that $gf = kh$ if and only if $f'g' = h'k'$.