18.727: Problem Set 7

Due: 4/18/01

1. Let $f\colon X \to Y$ be a continuous map. Show that the direct image functor f_* from the category of sheaves of abelian groups on X to that on Y is left exact and that there exists a spectral sequence

$$E_2^{p,q} = H^p(Y, R^q f_* F) \Rightarrow H^{p+q}(X, F).$$

This is the Leray spectral sequence. The sheafs $R^q f_* F$ are called the higher direct image sheafs of F.

2. Let F be a left exact functor, let M be an object, and let $M \to E^\cdot$ be a resolution of M by F-acyclic objects. Show that for all $q \geq 0$, $(R^q F)(M) = H^q(F(E^\cdot))$.

(*Hint*: Pick a fully injective resolution $E^\cdot \to I^{\cdot,\cdot}$ and consider the two spectral sequences associated with the double complex $F(I^{\cdot,\cdot})$.)