18.906: Problem Set 2

Due: Thursday, February 27.

1. Let X and Y be objects in a category $\mathcal{C}$, and let $*$ and $*'$ be two composition laws on the set of morphisms $\mathrm{Hom}_{\mathcal{C}}(X, Y)$. Assume that $*$ and $*'$ have a common two-sided identity element and are mutually distributive in the sense that

$$(f * f') *' (g * g') = (f *' g) * (f' *' g'),$$

for all $f, f', g, g' \in \mathrm{Hom}_{\mathcal{C}}(X, Y)$. Show that $*$ and $*'$ are equal, and that each is commutative and associative.

2. Let (X, e) be an H-space with multiplication $\mu\colon X \times X \to X$ (see Hatcher p. 281). Show that for all $n \geq 1$, the group structure on $\pi_n(X, e)$ defined by

$$(f * g)(x) = \mu(f(x), g(x))$$

is equal to the usual group structure. Show further that $\pi_1(X, e)$ is an abelian group.

3. Hatcher, chap. 4, §1, exercise 11.

4. Hatcher, chap. 4, §1, exercise 23.