18.917: *Topics in algebraic topology*

Time and place: TR 1-2:30 in 8-119.

In this course, I plan to give an introduction to topological Hochschild homology and the de Rham-Witt complex.

Associated to any ring A or, more generally, to any category with cofibrations and weak equivalences $\mathcal{C}$ one has the topological Hochschild spectrum $T(A)$ or $T(\mathcal{C})$, which is an object in the $\mathbb{T}$-stable homotopy category ($\mathbb{T}$ is the circle group). In the equivariant setting the natural notion of cells are (the suspension $\mathbb{T}$-spectra associated with) the $\mathbb{T}$-spaces $D^q \times \mathbb{T}/C$, where $C \subset \mathbb{T}$ is a closed subgroup. Hence, the natural notion of homotopy groups are maps in the homotopy category from

$$S^q \wedge \mathbb{T}/C_+ = (D^q \times \mathbb{T}/C)/(\partial D^q \times \mathbb{T}/C).$$

We fix a prime p and consider the equivariant homotopy groups

$$\mathrm{TR}_q^n(A;p) = [S^q \wedge \mathbb{T}/C_{p^{n-1}+}, T(A)]_{\mathbb{T}}.$$

There are usually very large abelian groups. But as n and q varies they are related by a number of operators and the combined algebraic structure is rather rigid. We call this structure a *Witt complex* over A. There is an initial example of this structure called the de Rham-Witt complex, and we use the canonical map

$$W_n\Omega_A^q \to \mathrm{TR}_q^n(A;p)$$

to describe the right-hand groups. This map is similar to the canonical map from Milnor K-theory to algebraic K-theory, and in favorable cases, the latter can be expressed in terms of the former.

After an introduction, here is a tentative route:

(i) Cyclic objects.

(ii) Equivariant stable homotopy theory.

(iii) The topological Hochschild spectrum.

(iv) The de Rham-Witt complex.

(v) The Tate spectrum.