Homotopy Theory

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Evaluation: Occational exercises reviewed by the teacher.

Object of the course: This course gives an introduction to homotopy theory. Classically, this is the study of the weak homotopy-type of topological spaces, a notion that goes back to Poincaré. A continuous map of topological spaces is called a *weak equivalence* if it induces an isomorphism of homotopy groups, and the *weak homotopy-type* of a topological space is the isomorphism class of the space in the category obtained by formally introducing an inverse map for every weak equivalence. Hence, it the structure of this category, the *homotopy category* of spaces, that is the main object of study. The main techniques are centered around two classes of maps called the *fibrations* and the *cofibrations* that were introduced by Serre and J. H. C. Whitehead, respectively. The properties of the category of topological spaces together with the three classes of maps given by the weak equivalences, the fibrations, and the cofibrations were formalized by Quillen into the notion of a *model category* for a homotopy theory. This facilitates the use of homotopy theoretical methods in other areas of mathematics, most prominently, the current work by Morel and Voevodsky on the homotopy theory of algebraic varieties.

Schedule of the course: We begin with the basic notions of a model category and associated model category. As an example we will consider the unbounded derived category of a ring. The further choice of topics and the pace of the course will depend on the participants. One possible conclusion to the course is the algebraic model of the rational homotopy theory of spaces and the proof of the rational homotopy type of a Kähler manifold is determined by the rational cohomology ring.

Keywords: Homotopy, model categories, fibrations, rational homotopy theory.

Required knowledge: An introductory course in algebraic topology including the fundamental group and covering spaces.