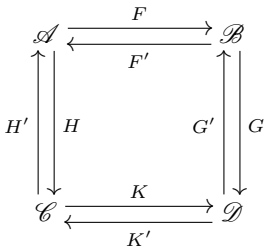
Problems for Recitation 6

There is only one problem this time, but it proves an extremely useful result.

Consider categories and functors



and assume that F (resp. G, resp. H, resp. K) is left adjoint to F' (resp. G', resp. H', resp. K') and that unit and counit maps of the adjunction have been chosen. Prove the following statements:

(i) A natural transformation $f\colon G \circ F \Rightarrow K \circ H$ determines and is determined by a natural transformation $f'\colon H' \circ K' \Rightarrow F' \circ G'$.

(ii) The natural transformation $f\colon G \circ F \Rightarrow K \circ H$ is an isomorphism if and only if the corresponding natural transformation $f'\colon H' \circ K' \Rightarrow F' \circ G'$ is an isomorphism.