$$
\begin{tikzcd}
H^{2c}_{Zar}(X, \mathbb{Z}(c)) \arrow[r, "cl"] & H^{2c}(X, \mathbb{Q}_\ell(c)) \ni cl(Y) \\
& H^{2c}(Y, \mathbb{Q}_\ell(c)) \ni [Y] \arrow[u, "i_*"] \\
& H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell) \ni 1 \arrow[u, "g_*", "\wr"']
\end{tikzcd}
$$

Thm let X be a connected scheme smooth rel. dim. d and proper over an algebraically closed field k, let l be a prime number not divisible by char(k), and let F be an endomorphism of X. If the fixed points of F are isolated, then their number, counted with multiplicity is equal to

$$\sum_{i=0}^{2d} (-1)^i \, \mathrm{Tr}\left(F^* \mid H^i(X, \mathbb{Q}_\ell)\right).$$

$$CH^i(X) \longrightarrow H^{2i}(X, \mathbb{Q}_\ell(i))$$

$$H^{2d}(X, \mathbb{Q}_\ell(d)) \xrightarrow[\sim]{f_*} H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell(0)) = \mathbb{Q}_\ell$$

$$H^i(X, \mathbb{Q}_\ell(j)) \otimes H^{2d-i}(X, \mathbb{Q}_\ell(d-j))$$

$$\xrightarrow{\cup} H^{2d}(X, \mathbb{Q}_\ell(d)) \xrightarrow{\sim} \mathbb{Q}_\ell \quad \text{perfect}$$

$$H^*(X, \mathbb{Q}_\ell) \otimes H^*(Y, \mathbb{Q}_\ell) \xrightarrow[\sim]{pr_1^* + pr_2^*} H^*(X \times Y, \mathbb{Q}_\ell)$$

$$H^{2d}(X \times X, \mathbb{Q}_\ell(d)) \xrightarrow{\sim} \mathrm{End}(H^*(X, \mathbb{Q}_\ell))$$

$$H^{2d}(X \times X, \mathbb{Q}_\ell(d)) \otimes H^*(X, \mathbb{Q}_\ell)$$

$$\xleftarrow{\sim} \bigoplus_{0 \le i \le 2d} H^i(X, \mathbb{Q}_\ell) \otimes H^{2d-i}(X, \mathbb{Q}_\ell(d)) \otimes H^*(X, \mathbb{Q}_\ell)$$

$$\longrightarrow \bigoplus_{0 \le i \le 2d} H^i(X, \mathbb{Q}_\ell) \otimes H^i(X, \mathbb{Q}_\ell)^\vee \otimes H^i(X, \mathbb{Q}_\ell)$$

$$\longrightarrow \bigoplus_{0 \le i \le 2d} H^i(X, \mathbb{Q}_\ell) = H^*(X, \mathbb{Q}_\ell)$$

$$\mathrm{End}(H^*(X, \mathbb{Q}_\ell)) = \mathrm{Hom}(H^*(X, \mathbb{Q}_\ell), H^*(X, \mathbb{Q}_\ell))$$

$$\xleftarrow{\sim} \bigoplus_{0 \le i \le 2d} H^i(X, \mathbb{Q}_\ell) \otimes H^i(X, \mathbb{Q}_\ell)^\vee$$

$$\xrightarrow{\gamma} \bigoplus_{0 \le i \le 2d} H^i(X, \mathbb{Q}_\ell)^\vee \otimes H^i(X, \mathbb{Q}_\ell)$$

$$\xrightarrow{\varepsilon} \mathbb{Q}_\ell$$

$$H^{2d}(X\times X, \mathbb{Q}_\ell(d)) \xrightarrow{\sim} \mathrm{End}(H^*(X, \mathbb{Q}_\ell))$$
$$cl(\Delta_X) \xmapsto{?} \mathrm{id}$$

$$\begin{tikzcd}
H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell) & \ni & 1 \\
H^{2d}(X, \mathbb{Q}_\ell(d)) \arrow[u, "\sim"', "f_*"] \arrow[d, "\Delta_{X*}"] & \ni & {[X]} \arrow[u, mapsto] \arrow[d, mapsto] \\
H^{2d}(X\times X, \mathbb{Q}_\ell(d)) & \ni & cl(\Delta_X)
\end{tikzcd}$$

$$\begin{tikzcd}
H^{2d}(X, \mathbb{Q}_\ell(d)) \otimes H^{2d}(X\times X, \mathbb{Q}_\ell(d)) \arrow[r, "\Delta_{X*}\otimes \mathrm{id}"] \arrow[d, "\mathrm{id}\otimes \Delta_X^*"] & H^{2d}(X\times X, \mathbb{Q}_\ell(d)) \otimes H^{2d}(X\times X, \mathbb{Q}_\ell(d)) \arrow[d, "\cup"] \\
H^{2d}(X, \mathbb{Q}_\ell(d)) \otimes H^{2d}(X, \mathbb{Q}_\ell(d)) \arrow[r, "pr_1^*\otimes pr_2^*"] \arrow[d, "f_*\otimes f_*"] & H^{4d}(X\times X, \mathbb{Q}_\ell(2d)) \arrow[d, "(f\times f)_*"] \\
H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell) \otimes H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell) \arrow[r, "\cup"] & H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell)
\end{tikzcd}$$

$$H^*(X\times X) \xrightarrow{\sim} \underline{\mathrm{End}}(H^*(X))(-d)[-2d]$$

$$H^*(X\times X)\otimes H^*(X)(d)[2d]$$

$$\xleftarrow{\sim} H^*(X)\otimes H^*(X)\otimes H^*(X)(d)[2d]$$

$$\xrightarrow{id\otimes b} H^*(X)\otimes H^*(X)\otimes H^*(X)^\vee$$

$$\xrightarrow{id\otimes\cdot\varepsilon} H^*(X)$$

In particular,

$$H^{2d}(X\times X)\xrightarrow{\sim} \mathrm{End}(H^*(X)).$$

Moreover,

$$\begin{tikzcd}
H^{2d}(X\times X)\otimes H^{2d}(X\times X) \arrow[r, "\sim"] \arrow[d, "\cup"] & \mathrm{End}(H^*(X))\otimes \mathrm{End}(H^*(X)) \arrow[d, "\circ"] \\
H^{4d}(X\times X) \arrow[d, "\mathrm{Tr}", "\sim"'] & \mathrm{End}(H^*(X)) \arrow[d, "\mathrm{Tr}"] \\
\mathbb{Q}_\ell \arrow[r, equal] & \mathbb{Q}_\ell
\end{tikzcd}$$

$$
\begin{tikzcd}
X \arrow[r, "\Delta_{X/k}"] \arrow[d, "f"] & X \times X \arrow[d, "f \times f"] \arrow[dl, dashed, "g"'] \\
\mathrm{Spec}(k) \arrow[r, "\Delta_{k/k}", "\sim"'] & \mathrm{Spec}(k) \times \mathrm{Spec}(k)
\end{tikzcd}
$$

$$g_*(\mathrm{cl}(\Delta_X) \cup u) = f_*(u)$$

for all $u \in H^{2d}(X \times X, \mathbb{Q}_\ell(d))$

$$\mathbb{Q}_\ell = H^0(\mathrm{Spec}(k), \mathbb{Q}_\ell) \xleftarrow[\sim]{f_*} H^{2d}(X, \mathbb{Q}_\ell(d))$$