Problems for Recitation 2

1. Let C be a category and let $(F_i)_{i \in I}$ be a family of presheaves on C. For every object X of C, define $J(X)$ to be the set of sieves S on X with the property that for every $i \in I$ and every morphism $f \colon Y \to X$ in C, the canonical map

$$F_i(Y) \longrightarrow \lim{}_{f^*(S)^{\mathrm{op}}}(F_i)_{Y, f^*(S)}$$

is a bijection. Show that J is a topology on C.

(If the family in question is the family $(h(X))_{X \in \mathrm{ob}(\mathsf{C})}$ of all representable presheaves on C, then the topology J is called the *canonical topology*.)

2. Let (C, J) and (C', J') be two sites and let (v, u, ϵ, η) be an adjunction from C' to C. Show that u is continuous if and only if v is cocontinuous. In this case, show that the functors u^s and v^*, or equivalently, the functors u_s and v_* are canonically naturally isomorphic. Conclude that the functor u^s preserves finite limits.