Algebra III/Introduction to Algebra III: Representation Theory

Due: Please upload solutions to NUCT by Tuesday, April 28, 2020.

Problem 1. (1) Let G be a group and let $\pi\colon G \to \mathbb{C}^\times$ be a one-dimensional complex representation of a group G. Show that if an element $g \in G$ has (finite) order n, then $\pi(g) \in \mathbb{C}^\times$ is an nth root of unity.

(2) Let G be a cyclic group of order n. Show that, up to isomorphism, G admits exactly n one-dimensional complex representations.

[Hint: First construct n one-dimensional complex representations $\pi_i\colon G \to \mathbb{C}^\times$, $0 \leq i < n$, such that $\pi_i \simeq \pi_j$ implies that $i = j$. Next show that if $\pi\colon G \to \mathbb{C}^\times$ is any one-dimensional complex representation, then $\pi \simeq \pi_i$ for some $0 \leq i < n$.]

Problem 2. Find, up to isomorphism, all one-dimensional complex representations of the infinite cyclic group $G = (\mathbb{Z}, +)$.