**Algebra III/Introduction to Algebra III: Representation Theory**

*Due*: Please upload solutions to NUCT by Tuesday, May 5, 2020.

**Problem 1.** Let $\pi\colon G \to GL(V)$ be a representation, let $U \subset V$ be a $\pi$-invariant subspace, and suppose that $W_1 \subset V$ and $W_2 \subset V$ are two $\pi$-invariant complements of $U$. Show that $\pi_{W_1}\colon G \to GL(W_1)$ and $\pi_{W_2}\colon G \to GL(W_2)$ are isomorphic.

**Problem 2.** Let $G = (\mathbb{C}, +)$ be a group of complex numbers under addition, let $A \in M_n(\mathbb{C})$ be a complex $n \times n$-matrix, and consider the representation

$$G \xrightarrow{\pi} GL(V)$$

on $V = \mathbb{C}^n$ given by $\pi(s) = e^{sA}$. Suppose that the characteristic polynomial[1]

$$\chi_A(t) = \det(A - tE)$$

has $n$ distinct roots $\lambda_1, \dots, \lambda_n$. Find all $\pi$-invariant subspaces of $V$.

[1] The definition of the characteristic polynomial has been corrected.