

Pespectives in Mathematical Sciences

Due: Tuesday, June 9, 2020, on NUCT.

Problem 1. Let \mathbb{H} be the division ring of quaternions defined Example 1.3 (4) of Lecture 1, and let $V = (\mathbb{R}^4, +, \cdot)$ be the left \mathbb{H} -vector space with “+” given by the usual vectorsum in \mathbb{R}^4 and with scalar multiplication $\cdot : \mathbb{H} \times \mathbb{R}^4 \rightarrow \mathbb{R}^4$ defined by

$$(a + ib + jc + kd) \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_3 \end{pmatrix}.$$

Prove the following statements:

- (a) The family (\mathbf{v}) consisting of the single vector

$$\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

is a linearly independent family in the left \mathbb{H} -vector space V .

- (b) The family (\mathbf{v}) generates the left \mathbb{H} -vector space V .
(c) The family (\mathbf{v}) is a basis left \mathbb{H} -vector space V .