

## Algebra I/Introduction to Algebra V

*Due:* Please upload solutions to NUCT by Tuesday, June 8, 2021.

**Problem 1.** Let  $(V, \pi)$  be a finite dimensional complex representation of a finite group  $G$ , let  $(V^*, \pi^*)$  be the dual representation, and let  $\chi_\pi, \chi_{\pi^*} : G \rightarrow \mathbb{C}$  be their characters.

- (a) Show that for all  $g \in G$ ,  $\chi_{\pi^*}(g) = \overline{\chi_\pi(g)}$ .
- (b) Show that  $\pi \simeq \pi^*$  if and only if  $\chi_\pi : G \rightarrow \mathbb{C}$  takes all its values in  $\mathbb{R} \subset \mathbb{C}$ .
- (c) Recall the two-sided regular representation  $(\mathbb{C}[G], \text{Reg})$  of  $G \times G$  defined by

$$\text{Reg}(g_1, g_2)(f)(x) = f(g_2^{-1}xg_1).$$

Show that  $\text{Reg} \simeq \text{Reg}^*$ .