

## Algebra I/Introduction to Algebra V: Representation Theory

*Due:* Please upload solutions to TACT by Tuesday, April 25, 2023.

**Problem 1.** Let  $\pi: G \rightarrow GL(V)$  be a representation, let  $U \subset V$  be a  $\pi$ -invariant subspace, and suppose that  $W_1 \subset V$  and  $W_2 \subset V$  are two  $\pi$ -invariant complements of  $U$ . Show that  $\pi_{W_1}: G \rightarrow GL(W_1)$  and  $\pi_{W_2}: G \rightarrow GL(W_2)$  are isomorphic.

[Hint: Show that both are canonically isomorphic to a common representation.]

**Problem 2.** Let  $G = (\mathbb{C}, +)$  be the group of complex numbers under addition, let  $A \in M_n(\mathbb{C})$  be a complex  $n \times n$ -matrix, and consider the representation

$$G \xrightarrow{\pi} GL(V)$$

on  $V = \mathbb{C}^n$  given by  $\pi(s) = e^{sA}$ . Suppose that the characteristic polynomial

$$\chi_A(t) = \det(A - tE)$$

has  $n$  distinct roots  $\lambda_1, \dots, \lambda_n$ . Find all  $\pi$ -invariant subspaces of  $V$ .

[Hint: There are  $2^n$  of them.]