

Algebra I/Introduction to Algebra V: Representation Theory

Due: Please upload solutions to TACT by Tuesday, May 2, 2023.

Problem 1. Let $\pi: G \rightarrow \mathrm{GL}(V)$ be a unitary representation of the group G on a finite dimensional complex vector space V . Prove that for all g , the eigenvalues of the linear automorphism $\pi(g): V \rightarrow V$ have absolute value 1.

Problem 2. Let $G = \{1, \zeta, \zeta^2\}$ be a cyclic group of order 3, and let $\pi: G \rightarrow \mathrm{GL}_2(\mathbb{R})$ be the representation of G on \mathbb{R}^2 defined by

$$\pi(\zeta) = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}.$$

Find an inner product on \mathbb{R}^2 that is invariant with respect to π in the sense that

$$\langle \pi(g)(\mathbf{x}), \pi(g)(\mathbf{y}) \rangle = \langle \mathbf{x}, \mathbf{y} \rangle$$

for all $g \in G$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$.