Geometry III/Introduction to Geometry III: Riemann Surfaces

Problem 1 (Riemann surface of a holomorphic function)**.** Let $f\colon U \to \mathbb{C}$ be a holomorphic function defined on a connected open subset of the complex plane. Let $|X(f)|$ to be the quotient of the set of pairs (γ, F) of a path $\gamma\colon [0,1] \to \mathbb{C}$ with $\gamma(0) \in U$ and an analytic continuation F of f along γ by the equivalence relation that identifies (γ, F) and (σ, G) if $\gamma(1) = \sigma(1)$ and if F and G agree in some open disc around the common point.

Show that, up to unique isomorphism, the set $|X(f)|$ admits a unique structure of Riemann surface with the property that the map

$$X(f) \xrightarrow{\;p\;} \mathbb{C}$$

that to the equivalence class of (γ, F) assigns $\gamma(1)$ is a local isomorphism.

(This Riemann surface is in a sense the largest domain on which f can live as a single-valued function.)