Abstracts for the Workshop on Extremes in Space and Time, May 31

On the extremal behavior of random variables observed at renewal times

Bojan Basrak (University of Zagreb)

This is joint work with Drago Špoljarić (University of Zagreb).

We consider the asymptotic extremal behavior of iid observations $X_1, X_2, \ldots$, until a random time $\tau(t)$ which is determined by a renewal process, possibly dependent on X_i's. The maximum of these observations

$$M(t) = \max_{i \le \tau(t)} X_i \,,$$

has been studied for decades. The first advances have already been made in the 1960s in the relatively straightforward case of a renewal process with finite mean interarrival times. Anderson [1] was the first to study the limiting behavior of $M(t)$ in the case of a renewal process with infinite mean interarrival times. More recently, his result has been extended to describe the limiting behavior of $(M(t))$ at the level of processes (see [2], for instance).

Using point processes techniques, we show how one can recover these known results and characterize the asymptotic behavior of the upper order statistics in the sequence X_i until time $\tau(t)$. We also allow for certain types of dependence between the observations and interarrival times, relaxing the conditions previously used in the literature. Finally, we show how our approach yields some well known and apparently new results concerning e.g. the longest run of heads and the maximal sojourn time of a continuous time random walk.

References

[1] Anderson, K.K. (1987). Limit theorems for general shock models with infinite mean intershock times. *J. Appl. Probab.*, **24**, 449–456.

[2] Meerschaert, M.M. and Stoev, S.A. (2009). Extremal limit theorems for observations separated by random power law waiting times. *J. Statist. Plann. Inference*, **139**, 2175–2188.

Large deviation estimates for exceedances of perpetuity sequences

Jeffrey Collamore (University of Copenhagen)

In a wide variety of problems in pure and applied probability, it is of interest to study the extremal events of perpetuity sequences. Estimates for the stationary tail distribution of perpetuity sequences have been developed in the classical papers of Kesten (1973) and Goldie (1991). It is well known that if $Y_n := B_1 + A_1 B_2 + (A_1 \cdots A_{n-1}) B_n$, then

$$\mathbf{P}\left\{ \lim_{n\to\infty} Y_n > u \right\} \sim C u^{-R} \quad \text{as} \quad u \to \infty$$

for certain constants C and R. Recently, it has been shown in Collamore and Vidyashankar (2013) that this stationary tail estimate may also be obtained via other methods, based roughly on large deviation theory and Markov chain theory. In this way, one obtains certain refinements, including an explicit representation for the constant C.

Following a description of some of these results, we then consider how large deviation methods may be used to go beyond this classical estimate and describe certain path properties of the perpetuity sequence. Our specific goal is to characterize the first passage time distribution into the set (u, ∞) and some related exceedance probabilities. We establish a large deviation principle for $T_u := \inf\{n : Y_n > u\}$ and provide some associated sharp asymptotic estimates. (Joint work with D. Buraczewski, E. Damek, and J. Zienkiewicz).

Hidden regular variation in joint tail modeling with likelihood inference via the MCEM algorithm

Dan Cooley (Colorado State University)

This is joint work with Grant Weller, Department of Statistics, Colorado State University.

A fundamental deficiency of classical multivariate extreme value theory is the inability to distinguish between asymptotic independence and exact independence. In this work, we examine multivariate threshold modeling based on the framework of regular variation on cones. Tail dependence is described by a limiting measure, which in some cases is degenerate on joint tail regions despite strong sub-asymptotic dependence in such regions. Hidden regular variation, a higher-order tail decay on these regions, offers a refinement of the classical theory. This work develops a representation of random vectors possessing hidden regular variation as the sum of independent regular varying components. The representation is shown to be asymptotically valid via a multivariate tail

equivalence result, and an example is demonstrated via simulation. We develop a likelihood-based estimation procedure from this representation via a version of the Monte Carlo expectationmaximization algorithm which has been modified for tail estimation. The methodology is demonstrated on simulated data and applied to a bivariate series of air pollution data from Leeds, UK. We demonstrate the improvement in tail risk estimates offered by the sum representation over approaches which ignore hidden regular variation in the data.

Local robust and asymptotically unbiased estimation of conditional Pareto-type tails

Yuri Goegebeur (University of Southern Denmark Odense)

This is joint work with Goedele Dierckx (Brussels) and Armelle Guillou (Strasbourg & CNRS).

We introduce a nonparametric robust and asymptotically unbiased estimator for the tail index of a conditional Pareto-type response distribution in presence of random covariates. The estimator is obtained from local fits of the extended Pareto distribution to the relative excesses over a high threshold using an adjusted minimum density power divergence estimation technique. We derive the asymptotic properties of the proposed estimator under some mild regularity conditions, and also investigate its finite sample performance with a small simulation experiment.

Efficient importance sampling in a credit risk model with contagion

Henrik Hult (KTH Stockholm)

This talk is based on joint work with Pierre Nyquist.

Consider a credit portfolio consisting of n obligors divided into d homogeneous groups. The time evolution of the number of defaults in each group can be modelled as a continuous time pure jump Markov chain. We will be interested in computing the probability that a large fraction of obligors default within a fixed time horizon. In a recent paper R. Carmona and S. Crepey (Int. J. Theor. Appl. Finance, 13, 2010) considered an importance sampling algorithm for computing loss probabilities in this model. Their algorithm performs well when there is no contagion effect, but poorly in the presence of contagion. We will demonstrate how the subsolution approach by P. Dupuis and H. Wang can be adopted in order to construct efficient importance sampling algorithms in the presence of contagion. The key step is to find an appropriate subsolution to an associated partial differential equation of Hamilton-Jacobi type.

Asymptotic independence of stochastic volatility models

Anja Janssen (University of Hamburg)

This is joint work with Holger Drees (University of Hamburg)

Discrete-time stochastic volatility (SV) models have become a standard tool for the modeling of economic time series as they are able to reflect many of the well-known stylized facts of financial markets. With regard to their extremal behavior, the standard SV model specifications have in common that consecutive observations are asymptotically independent and thus their extremal index is equal to 1.

However, on a pre-asymptotic level SV models may still show a clustering of large values and we are therefore interested in the second order behavior of extremal dependence. Different concepts may be applied which allow for a finer analysis of asymptotic independence: See, in particular, the coefficient of tail dependence proposed by [1] and the notion of hidden regular variation, cf. [2]. However, the standard model specifications for SV models do not reflect the broad spectrum of possible second order behavior since their asymptotic properties are mainly determined by the heavy-tailed i.i.d. innovations. With a view towards empirical results for real life processes we suggest an alternative model which allows for more freedom in the asymptotic dependence structure. We analyze this model in the framework of hidden regular variation under the use of suitable general results.

References

[1] Ledford, A.W. and Tawn, J.A. Statistics for near independence in multivariate extreme values. *Biometrika* (83):169–187, 1996.

[2] Resnick, S.I. Hidden regular variation, second order regular variation and asymptotic independence. *Extremes* (5):303–336, 2002.

Rare-event analysis and Monte Carlo methods for Gaussian processes

Jingchen Liu (Columbia University)

Gaussian processes are employed to model spatially varying errors in various stochastic systems. In this talk, we conduct an analysis of the extreme behavior and related rare-event simulation problems for such systems. In particular, the topic covers various nonlinear functionals of Gaussian processes, including the supremum norm and integral of convex functions. We present asymptotic results and efficient simulation algorithms for the associated rare-event probabilities.

Solvency Capital Requirement calculation for different hedge funds strategies

Olivier Wintenberger (Paris Dauphine)

The Solvency II directive provides a standard formula to calculate similarly the SCR for other equities such as any Hedge fund strategies. We propose an alternative internal model relying on quantitative risk management. The SCR is computed using an estimation of the cluster index. This new methodology differentiate the strategies at the price of a more complicated calculation.

Frequency analysis of heavy tail phenomena

Yuwei Zhao (University of Copenhagen)

This is joint work with Thomas Mikosch (Copenhagen)

Heavy tail phenomena have been observed in various fields such as telecommunication networks (Internet), insurance, finance, seismology, to name a few. It is typical for these phenomena that they are rare and serially dependent. The tools of classical time series analysis (autocorrelation function, spectral density) are not suitable for describing extreme events and their dependence structure. However, in our approach we adapt the autocorrelation function from the time domain and the spectral density from the corresponding frequency domain of time series analysis: we apply them to indicator functions of the rare events of interest. Thus we deal with stationary sequences of indicator functions whose distributions change in dependence on a sufficiently high threshold. Therefore the classical results of time series analysis are not directly applicable: one deals with a triangular array of stationary sequences whose marginal distribution changes with the threshold. First, we introduce the extremogram as analog of the autocorrelation function of a stationary sequence and its sample version based on counts of rare events in a stationary sequence. Second, we define a corresponding periodogram, called *ex-periodogram* as an estimator of the spectral density defined via the extremogram. The ex-periodogram shares various of the classical properties of the periodogram of a stationary weakly dependent sequence: the ex-periodogram ordinates at distinct (Fourier) frequencies are asymptotically independent and exponentially distributed and smoothed versions are consistent estimators of the spectral density. Having established some basic asymptotic theory for the ex-periodogram our next goal is to apply the theory for estimating the spectral distribution function, parameter estimation of suitable time series models and goodness of fit tests. In this context, we study different versions of the integrated ex-periodogram. It is our objective to derive results parallel to the classical theory for the integrated periodogram of a stationary sequence which can be interpreted as a spectral empirical distribution. We show that the integrated ex-periodogram satisfies a functional central limit theorem under mild conditions, which ensure that goodness-of-fit

tests, such as the Grenander and Rosenblatt and the Cramér-von Mises tests, can be constructed for the ex-periodogram. This means that we can determine whether a model is suitable solely based on the behavior of extreme events.