Equiv Eues & Dowling lattices

1. Equiv Eues
2. Gen'l defn of Dowling latt
3. 3 example
4. Equiv Eue of Dowling lattices.

Atiyah-Segal   3SJ ABC

① $\Pi \circlearrowleft G$ $r \geq 1$ $\tilde{\chi}_r(\Pi, G) = \frac{1}{|G|} \sum_{x \in Hom(\mathbb{Z}^r, G)} \tilde{\chi}(C_\Pi(x))$

$\tilde{\chi}_1(\Pi, G) = \tilde{\chi}(|\Pi|/G) \in \mathbb{Z}$

$\tilde{\chi}_2(\Pi, G) = \dim K^0_G(|\Pi|, \phi) - \dim K^1_G(|\Pi|, \phi) \in \mathbb{Z}$ (Atiyah-Segal)

$\boxed{KRC : Z_p(G) + \tilde{\chi}_2(\mathcal{P}^{\neq \varepsilon}_{G}, G) = 0}$

$\tilde{\chi}_3(\Pi, G) = ?$

Recursion : $\tilde{\chi}_{r+1}(\Pi, G) = \sum_G \tilde{\chi}_r(C_\Pi(a), C_G(a)) \in \mathbb{Z}$

To compute : Vanishing thm + a recursion

Examples ① $B(n) = 2^n$ subsets of $n = \{1, .., n\} \circlearrowleft \Sigma_n$

$$\sum \tilde{\chi}_r(B(n)^*, \Sigma_n) x^n = \prod (1 - x^d)^{\lambda_r(d)}$$

$\lambda_r(d)$ ~~$r(d)$~~ $= \#$ index d subgroups of $\mathbb{Z}^r$

② $\Pi(n)$ = set partition of $\{1, .., n\} \circlearrowleft \Sigma_n$

Dirichlet convolution : $(f * g)(d) =$

Defn $(c_r * \lambda_r)(n) = (-1)^{n+1}$ – this defines $c_r(n)$

$$\tilde{\chi}_r(\Pi(n), \Sigma_n) = \frac{1}{n} c_r(n)$$

③ q prime power

$L_n(\mathbb{F}_q)$ = subspaces of $\mathbb{F}_q^n \circlearrowleft GL_n(\mathbb{F}_q)$

$$\sum \tilde{\chi}_{r+1}(L_n(\mathbb{F}_q)^*, GL_n(\mathbb{F}_q))x^n = \exp\left(-\sum (q^n-1)^r \frac{x^n}{n}\right)$$

Significance ?

$\tilde{\chi}_2(L_n(\mathbb{F}_q)^*, GL_n(\mathbb{F}_q)) = q-1$ proves Knörr-Robinson conj ~~confirms~~ AWC for $GL_n(\mathbb{F}_q)$

~~for~~ at the ~~char.~~ gen. char.

④ $L_n^-(\mathbb{F}_q) \circlearrowleft GL_n^-(\mathbb{F}_q)$ $\langle\ ,\ \rangle$ over q^2 — proves Knörr-Robinson conj

Similar formula, numbers seen before in algebraic geometry

⑤ $Sp_n(\mathbb{F}_q)$ To the referee: Each case uses previous case

Conjecture: $\tilde{\chi}_r(\mathcal{S}^*_{\Sigma_n}, \Sigma_n) = (-1)^{n+1} 2^{r-1} |\mathrm{Out}(\Sigma_n)|$

also: where $(1,2)^{\Sigma_n}$ (on conjugacy class)

is it contractible?

Gen'l defn of Dowling lattices (like you have never seen before)

$\mathcal{C}$ EI-cat $\quad [\mathcal{C}]$ is a poset : $[a] \le [b] \Leftrightarrow \mathcal{C}(a,b) \neq \emptyset$

$a \in Ob(\mathcal{C})$ $\quad$ For $a/\mathcal{C}$ = $\mathcal{C}$-morphism with domain a

$[a/\mathcal{C}]$ is a $\mathcal{C}(a,a)$-poset : Dowling poset of $a \in \mathcal{C}$.

<u>Example 1</u> $\quad \mathcal{C}$ = finite sets with surjective

$n \in \mathcal{C} \quad \mathcal{C}(n) = \Sigma_n \quad [n/\mathcal{C}]$ = poset of set partitions of n

$\hat{0} = n \xrightarrow{=} n \qquad \hat{1} = n \twoheadrightarrow 1$

$$\begin{tikzcd} n \arrow[r, two heads] \arrow[d, two heads] \arrow[dr, phantom, "\Gamma"] & k_2 \arrow[d] \\ k_1 \arrow[r, two heads] & k_1 \vee k_2 \end{tikzcd}$$

Dowling poset $= \Pi(n)$

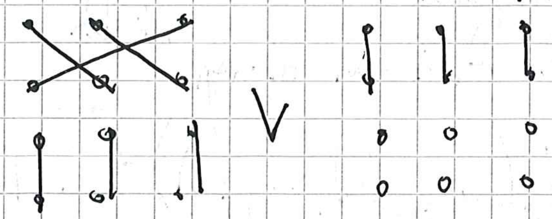
<u>Example 2</u> $\quad G$ finite group $\quad \bar{\mathcal{C}}$ = finite free G-sets with G-surjections

$nG \in Ob(\mathcal{C}) \quad \mathcal{C}(nG, nG) = G \wr \Sigma_n$

$\bar{D}_n(G) = [nG/\mathcal{C}] \circlearrowleft G \wr \Sigma_n \qquad$ Axel Hultman ABC 48

$\hat{0} = nG \to nG \qquad \hat{1} = nG \to 1G$ not unique defined !

~~What~~ The objects :



phylogenetic trees

$n = 4$

$G = 3$

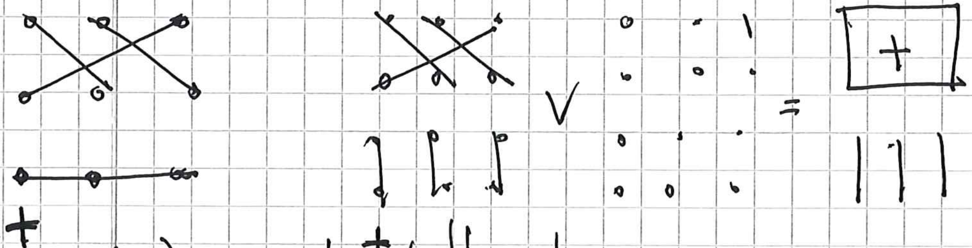
$\bar{D}_n(G) \to \Pi(n) \qquad \bar{D}_n(G)$ is a Gro. construction over $\Pi(n)$

$D_n^+(G)$ are all the same as posets

Example 3 $\mathcal{C}^+$ = finite free based G-sets with surjective G-maps

$nG_+ \in Ob(\mathcal{C})$ $\quad \mathcal{C}(nG_+, nG_+) = G\wr\Sigma_n$

$\hat{0} = nG_+ \xrightarrow{=} nG_+$ $\quad \hat{1} = nG_+ \to +$



$G = C_2$

$D_n^+(C_2)$ = intersection lattice for B_n

$D_n^+(G)$: Dowling lattice of G

$D_n^+(G) \to \Pi(n_+)$ $\quad D_n^+(G)$ is a modified Gr. construction over $\Pi(n_+)$

$D_n^+(G)$ is a graded atomic lattice

Vanishing Thm $\quad A \leq G\wr\Sigma_n = \mathcal{C}^\pm(nG_\pm)$

If A contains a normal subgroup B nontrivial with $(|B|,|G|)=1$ then

$C_{D_n^\pm(G)}(A)$ is ~~[illegible]~~ $C_{G\wr\Sigma_n}(A)$-contractible

The implication $\Leftarrow$ is unknown.

So Conj $\quad \tilde{\chi}_{r+1}(D_n^\pm(G), G\wr\Sigma_n)$ depends only on $|G|$

$G = C_m$, m prime $\quad a$

Recall: $\quad \chi_{r+1}(D_n^\pm(G), G\wr\Sigma_n) = \sum_{a}^{G} \chi_r(C_D(a), C(a))$

a is an m-elt

Thm $\prod_{\pm} \tilde{\chi}_{r+1}(D_n^{\pm}(G), G \wr \Sigma_n) = \binom{\pm m^r}{n}$ $\quad m = |G|$

Some kind of duality?

$G = C_1$ and $D_n^-(G)$ differs wildly from $G = C_1$ and $|G| > 1$

$\sum \tilde{\chi}_r(\Pi(n), \Sigma_n) x^n =$

$\sum \tilde{\chi}_r(\Pi(n), \Sigma_n) n x^n = \sum c_r(n) x^n$

$\prod D_n^+(G) \rightleftarrows \Pi(n_+)$ should say something about the ues