Master Class on von Neumann algebras and groups
Copenhagen, January 25-29, 2010
Abstracts of lectures

Erik Christensen's lecture is titled *On a fixed point theorem for von Neumann algebras and some applications*. An abstract follows below:

In 1966 both R. V. Kadison and S. Sakai published a proof of the result that any derivation δ of a von Neumann algebra $\mathcal{M} \subseteq \mathcal{B}(H)$ into itself is inner, i.e., it has the form $\delta(m) = [a, m] = am - ma$, for some fixed a in $\mathcal{M}$. Both articles first show that there exists an operator y in $\mathcal{B}(H)$, such that $\delta(m) = [y, m]$. It took both authors some trouble to be able to replace the general operator y by an element a from $\mathcal{M}$. The arguments, upon which they based their proofs of this last step, are in essence the same, and may be considered as a fixed point result. I think that their method may be applicable to many other settings. I will first tell about the basic results by Kadison and Sakai, and then tell how their method has been changed by Allan Sinclair and myself, such that it can be used in more general settings. I will give two applications. The first gives - yet another - characterization of injective von Neumann algebras. The second shows that the so-called completely bounded Hochschild cohomology of a von Neumann algebra with coefficients in itself vanishes. Hopefully someone in the audience may be able to cook up - yet another - application. Have a try.

Uffe Haagerup's lectures are meant to provide a solid introductory background for the upcoming workshop on von Neumann algebras and group actions, and will cover a number of fundamental results, as follows:

The first lecture will address basic facts about II_1-factors, including the fundamental group of a II_1-factor, and construction of II_1-factors both from the left regular representation of ICC groups (the group von Neumann algebra construction), and from the *group - measure space* construction.

The next lecture will focus on a number of properties of a discrete groups G, which can be expressed in terms of properties of its group von Neumann algebra $L(G)$, namely amenability, weak amenability, exactness, Property H and property T, followed by a discussion of examples and open problems.

The last two lectures will be used to give an outline of Popa's original construction of II_1-factors with trivial fundamental group, the main example being $L(G)$ for $G = \mathbb{Z}^2 \rtimes SL(2, \mathbb{Z})$ (semi-direct product). This will include discussions of the following topics:

- Relative property H and relative property T for inclusions of groups and inclusions of von Neumann algebras.
- Gaboriau's cost of an equivalence relation, or, alternatively, Gaboriau's L^2-Betti numbers for an equivalence relation.

David Kyed's lecture will be an introduction to quantum groups, using the operator algebraic framework, with special focus on the so-called compact quantum groups. Despite the appearance of the word "quantum" in the title, the talk will require no prerequisites from physics - a bit of operator algebras is sufficient!

Magdalena Musat's lectures will address a number of problems in operator algebras, motivated by topics of current research in operator space theory and noncommutative probability, based on recent results obtained in collaboration with Uffe Haagerup. They will revolve around the following topics:

- Problems concerning the relationship between the structure of von Neumann algebras (more precisely, their type) and embedding properties of their preduals (viewed as operator spaces).
- Noncommutative Khintchine and Grothendieck inequalities.
- Factorization and dilation properties for completely positive maps on von Neumann algebras.