Corrections, clarifications, and up-dates to "An Introduction to K-theory for C^*-algebras"

Necessary corrections:

Page 5, l. 5: Replace "one and only one *-homomorphism ..." by "one and only one unital *-homomorphism ...".

Page 7, item (ii): Replace "X is compact" by "X is compact and metrizable".

Page 7, item (iv): This statement has to be modified when X and Y are not compact and when φ is not unital. In the first statement, one must insert the word "proper" in front of "continuous function" two places. (A continuous function $f\colon Y \to X$ is said to be *proper* if the preimage, $f^{-1}(K)$, of any compact subset K of X is compact.) In the second statement one must require that the image of φ is not a proper ideal of $C_0(X)$.

Page 11, l. -9: Replace "for all x, y in A, and..." with "for all x, y in $\widetilde{A}$, and...".

Page 14, Exercise 1.14: Replace "if and only if" with "if".

Page 19, l. 12: Replace "*retract*" by "*deformation retract*".

Page 20, l. 4: Replace "each bounded subset Ω of A" by "each bounded subset Ω of A^+".

Page 41, l. -2: Replace "a collection of maps $\varphi \mapsto F(\varphi)$" by "a map $\varphi \mapsto F(\varphi)$"

Page 46, 3.3.1: One should assume that the C^*-algebra A is unital; at least in the last section, where $K_0(\tau)$ is defined.

Page 53, l. 9: Change "Example 3.3.5" to "Example 3.3.4".

Page 55, Exercise 3.4 (iii): Delete "over $C(X)$" from "rectangular matrices $v_1, v_2, \dots, v_r$ over $C(X)$ such that..."

Page 74, l. 15: Replace "Use (iii) to show..." with "Use (iv) to show...".

Page 83, l. 14: After "for each pair of *commuting* elements a, b in A^+" add ", and such that τ extends (possibly in a non-canonical way) to a continuous function $\tau\colon M_2(A)^+ \to \mathbb{R}^+$ with the same properties."

Page 84, l. 6-7: Replace "every unital, stably finite, separable, exact C^*-algebra admits a faithful trace." by "every unital, stably finite, exact C^*-algebra admits a tracial state."

Page 94, Proposition 6.2.4 (iii): This statement should read:

$$\operatorname{Ker}(\mu_n) = \{a \in A_n : \lim_{m\to\infty} \|\varphi_{m,n}(a)\| = 0\}.$$

Page 100, l. -4: Replace "By Proposition 6.4.2 (iii)" by "By Proposition 6.4.2 (ii)".

Page 102, l. -2: Replace "$K_0(\varphi_{n,1})\colon K_0(A_n) \to K_0(M_n(A))$" with "$K_0(\varphi_{n,1})\colon K_0(A) \to K_0(M_n(A))$".

Page 103, l. 4: Replace "...$= K_0(g') = g$," with "...$= K_0(\kappa_n)(g') = g$,".

Page 103, l. 12–13: Replace "[31, Section 3.3]" with "[31, Section 3.4]".

Page 152, Exercise 8.18 (iv): Replace "a" by "c" in the second sentence "Show that there is an invertible element b in A with $[b]_1 = [\underline{a}]_1$ in $K_1(A)$."

Page 155, l. 11: Replace "and p in $\mathcal{U}_{2(n_1+n_2)}(\widetilde{I})$ by" with "and p in $\mathcal{P}_{2(n_1+n_2)}(\widetilde{I})$ by".

Page 167, l. -11: Replace "an isomorphism" with "injective" in "Moreover, φ is an isomorphism if and only if v..."

Page 169, Eq. (9.13): Move the minus sign appearing in the matrix so that the equation reads:

$$v = \begin{pmatrix} a & -(1-aa^*)^{1/2} \\ (1-a^*a)^{1/2} & a^* \end{pmatrix}$$

Page 188, l. 14: Correct "The inclusion $\mathrm{GL}_n(\widetilde{SA}) \subseteq \mathcal{U}_n(\widetilde{SA})$ is a π_0-equivalence..." to "The inclusion $\mathcal{U}_n(\widetilde{SA}) \subseteq \mathrm{GL}_n(\widetilde{SA})$ is a π_0-equivalence..."

Page 188, l. 15: Replace "$\mathcal{U}_n(\widetilde{SA}) \subseteq \mathrm{Inv}_0(n)$ is also..." with "$\mathrm{GL}_n(\widetilde{SA}) \subseteq \mathrm{Inv}_0(n)$ is also...".

Page 195, l. -9: Replace two occurances of "1_A" with "1_B".

Page 196, l. 9-10: Correct "the inclusion $\mathcal{P}_n(A) \subseteq \mathrm{GI}_n(A)$ is a π_0-equivalence:" to "the inclusion $\mathrm{GI}_n(A) \subseteq \mathcal{P}_n(A)$ is a π_0-equivalence:".

Page 203, l. 6: Replace "retract" by "deformation retract".

Page 203, l. 14–15: It is not true that

$$\{u \in C([0,1], \mathcal{V}(A)) : u(0) = u(1) = 1\} = \{u \in \mathcal{U}_\infty(\widetilde{SA}) : s(u) = 1\},$$

but $\{u \in \mathcal{U}_\infty(\widetilde{SA}) : s(u) = 1\}$ is a dense subset of $\{u \in C([0,1], \mathcal{V}(A)) : u(0) = u(1) = 1\}$, and this inclusion is a π_0-equivalence (which justifies the claim in line 17). (Use density and the fact that if u, v are elements of either set and if $\|u - v\| < 1$, then $u \sim_h v$ in the respective set, to show that the inclusion is a π_0-equivalence.)

Page 213, l. -7: Remove the equation

$$s\Big(v(t)\begin{pmatrix} z(t)^* & 0 \\ 0 & z(t) \end{pmatrix}\Big) = 1_{2n}.$$

(This equation does not make sense because we are not working in a C^*-algebra with an adjoined unit, and the equation is not needed for anything.)

Page 217, last line of Exercise 12.5: Replace "$\gamma_0 = \delta_0 \circ \beta_B$" with "$\gamma_0 = K_1(\iota) \circ \beta_B$", and "$\gamma_1 = \delta_1 \circ \theta_B$" with "$\gamma_1 = K_0(\iota) \circ \theta_B$".

Page 220, Eq. (13.1): Replace two occurances of the interval "$[0, 2\pi]$" by "$[0, 1]$".

<u>Clarifications and minor corrections:</u>

Page 2, l. -1: Insert "(a quotient of)" in front of "A as a vector space...".

Page 3, l. 2: After "... to this inner product." add the following text "(It requires extra work to make φ injective, and this is often done by taking the infinite direct sum of all such Hilbert spaces, one Hilbert space for each positive linear functional on A.)".

Page 9, l. -2: Remove one "the".

Page 17: After line 7 add "because the unitary on the left-hand side has spectrum $\{-1, 1\} \subsetneq \mathbb{T}$."

Page 22, l. 10: Replace this line by "$|\alpha| = 1$, that u is unitary, and that $q = upu^*$."

Page 35, l. -4: After "Let $(S, +)$ be an Abelian semigroup" add ", not necessarily with a neutral element. (We have chosen to work in this generality although the semigroups we shall consider actually do have a neutral element)."

Page 36, l. 9: After "It is called the *Grothendieck map*." add "If S has a neutral element 0, then γ_S is given by the simpler formula $\gamma_S(x) = \langle x, 0 \rangle$."

Page 56, Exercise 3.8: The given formulae for p is correct, but the example below is better. Replace the first sentence of the exercise with the following: "Show that an element p in $M_2(\mathbb{C})$ is a one-dimensional projection if and only if

$$p = \begin{pmatrix} (1+x)/2 & (y+iz)/2 \\ (y-iz)/2 & (1-x)/2 \end{pmatrix},$$

where $(x, y, z) \in \mathbb{R}^3$ satisfy $x^2 + y^2 + z^2 = 1$."

Page 67, l. -11: To see the second last equality, use Lemma 4.3.1 (ii).

Page 68, l. 8: Most standard text books on algebraic topology contain the *Five Lemma*; see for example (14.7) in M. J. Greenberg and J. R. Harper *Algebraic topology*, Addison–Wesley, 1981.

Page 137, l. 16: After "Lemma 2.1.3 (ii)" add "(or Corollary 2.1.4)".

Page 140, l. 13: Replace Lemma 8.2.3 (i) by the slightly more precise: "there is a unitary u in $\mathcal{U}_n(\widetilde{A})$ for some n such that $g = [u]_1$ and $\widetilde{\varphi}(u) \sim_h 1$ in $\mathcal{U}_n(\widetilde{B})$,"

Page 177, l. -13: Before "Each element g in ..." add "The two unitaries u and v referred to in the theorem do exist as we shall proceed to show."

Page 177, l. -10: After "Lemma 2.1.3 (ii)" add "(or Corollary 2.1.4)".

Page 198, l. 18: Use the Whitehead Lemma (Lemma 2.1.5) and its proof to see the last homotopy.