Richard Kadison and his mathematical legacy—A memorial conference

The Carlsberg Academy, November 29–30, 2019

Titles and abstracts

Christian Skau: *Keeping the field alive—reflections on Kadison's pivotal role*

As a Ph.D. student at the University of Pennsylvania (Penn) I followed a course given by Kadison on operator algebras (both C^*- and W^*-algebras). The course stretched over 4 consecutive semesters starting the Fall term 1970 and ending the Spring term 1972. The course gave a survey of the state of the art of operator algebra theory—to which Kadison had made major contributions—roughly speaking up until the revolutionary new developments occurring around that time. (For this, let Tomita-Takesaki theory and Connes be keywords as far as W^*-algebras are concerned, and AF-algebras and Bratteli/Elliott be keywords regarding C^*-algebra theory.)

All this gives a unique vantage point to put things into perspective and to remind us of Kadison's crucial role in advancing operator algebra theory.

Joachim Cuntz: *Endomorphisms — old and not so old.*

We describe C^*-algebras associated with endomorphisms of compact abelian groups.

Sergio Doplicher: *Quantum Spacetime and Planck Scales*

The Principles of Quantum Mechanics and of General Relativity indicate that Spacetime in the small (Planck scale) ought to be described by a noncommutative C^*-algebra, implementing spacetime uncertainty relations. A model C^*-algebra of Quantum Spacetime and its Quantum Geometry is described. Interacting Quantum Field Theory on such a background is discussed, with open problems and recent progress. Applications to cosmology suggest that the Planck scale ought to depend upon dynamics, and possible consequences in the large of the quantum structure in the small are outlined.

Siegfried Echterhoff: *The weak containment problem*

The weak containment problem asks, whether for an action of a locally compact group G on a C^*-algebra A, the full and reduced crossed products coincide if and only if G acts amenably on A. If G is discrete and exact, a positive answer was obtained by Matsumura in case where $A = C(X)$ for a compact G-space X.

In this lecture we extend Matsumura's result to actions of locally compact groups on arbitrary locally compact spaces by showing that for second countable G and X, with G

exact, the full and reduced crossed products coincide if and only if the action of G on X is measure-wise amenable. On the way we discuss various versions of amenability for group actions on C^*-algebras A and show that many of them coincide if G is exact and $A = C_0(X)$ is commutative.

This is work in progress with Alcides Buss and Rufus Willett.

Ilijas Farah: *Coarse spaces and uniform Roe algebras*

To a metric space X one can associate the algebra of finite propagation operators on $\ell_2(X)$. The norm-closure of this algebra is the uniform Roe algebra associated with X. How much information about X can be recovered from the associated uniform Roe algebra? In this talk I will survey recent results on this question and its variations. This is an ongoing joint work with Bruno de Mendonca Braga and Alessandro Vignati.

Cyril Houdayer: *Stationary actions of higher rank lattices on von Neumann algebras*

I will talk about a recent joint work with Remi Boutonnet in which we show that for higher rank lattices (e.g., $\mathrm{SL}(3,\mathbb{Z})$), the left regular representation is weakly contained in any weakly mixing unitary representation. This strengthens Margulis' normal subgroup theorem (1978), Stuck–Zimmer's stabilizer rigidity result (1992) as well as Peterson's character rigidity result (2014). We also prove that Uniformly Recurrent Subgroups (URS) of higher rank lattices are finite, answering a question of Glasner–Weiss (2014). The main novelty of our work is a structure theorem for stationary actions of higher rank lattices on von Neumann algebras.

Arthur Jaffe: *The Feldverein*

As a student, I learned about the "Field Club". That was how Res Jost referred to a group of friends—including Kadison, Wightman, Haag, Borchers, and Gårding—as well as many of their collaborators and students. I will talk about some questions raised by those pioneers, which were close to the heart of Dick Kadison.

Tim de Laat: *Exotic group C^*-algebras of simple Lie groups with real rank one*

It is well known that the universal and the reduced group C^*-algebra of a locally compact group coincide if and only if the group is amenable. In general, there can be many C^*-algebras, called exotic group C^*-algebras, which lie between these two algebras. In a joint work with Timo Siebenand, we consider simple Lie groups G with real rank one and investigate their exotic group C^*-algebras $C^*_{L^{p+}}(G)$, which are defined through L^p-integrability properties of matrix coefficients of unitary representations of G. For (almost) every connected simple Lie group with real rank one and finite center and $2 \leq q < p \leq \infty$, we determine when the canonical quotient map $C^*_{L^{p+}}(G) \twoheadrightarrow C^*_{L^{q+}}(G)$ has non-trivial kernel. To this end, it suffices to study the integrability properties of spherical functions of class one representations of G. Our results generalize recent results of Samei and Wiersma on exotic group C^*-algebras of $\mathrm{SO}_0(n,1)$ and $\mathrm{SU}(n,1)$, with completely different methods. If time permits, I will explain how the same strategy can be used in order to determine exotic group C^*-algebras of automorphism groups of trees.

Sorin Popa: *The ubiquitous hyperfine II_1 factor*

The hyperfinite II_1 factor R played a central role in operator algebras ever since Murray and von Neumann introduced it in 1943. It is in some sense the "smallest" infinite dimensional factor, as it can be embedded in any other such factor, and any ∞-dim subfactor of R is isomorphic to R (Connes 1976). Fuglede and Kadison have shown in 1951 that any II_1 factor M contains a copy of R that fails the bicommutant property. I will first discuss two recent results I obtained about R-embeddings: (1) Any separable non-type I factor admits an ergodic R-embedding; (2) any separable II_1 factor M admits a coarse R-embedding. I'll then explain the importance of studying pairs of R-embeddings, $R_0, R_1 \subset M$, with control of the type of $R_0 \vee R_1^{op}$ in $\mathcal{B}(L^2M)$. Finally, I'll discuss a conjecture stating that if M is stably single generated, then there exists an R-pair $R_0, R_1 \subset M$ with $R_0 \vee R_1$ properly infinite. If true, this would imply $L(\mathbb{F}_\infty)$ is ∞-generated and $L(\mathbb{F}_n)$, $2 \leq n \leq \infty$, are non-isomorphic.

Hannes Thiel: *C^*-algebras of stable rank one and their Cuntz semigroups*

The Cuntz semigroup is a geometric refinement of K-theory that plays an important role in the structure theory of C^*-algebras. It is defined analogously to the Murray-von Neumann semigroup by using equivalence classes of positive elements instead of projections.

We show that Cuntz semigroups of stable rank one C^*-algebras satisfy Riesz interpolation. If the C^*-algebra is also separable, it follows that its Cuntz semigroup has finite infima. This allows us to apply methods from lattice and semigroup theory to prove structural results about C^*-algebras. In particular, we solve three long-standing problems for C^*-algebras of stable rank one: the Blackadar–Handelman conjecture, the Global Glimm Halving problem, and the non-simple ranks problem.

Andreas Thom: *The Murray-von Neumann algebra and the unitary group of a II_1-factor*

The unitary group of a II_1-factor has been of major interest eversince it first appeared in the work of Murray and von Neumann. The naturally associated Lie algebra can be identified with the skew-adjoint part of the Murray-von Neumann algebra, another natural object associated with a II_1-factor. Kadison has contributed to their understanding already in his early work in the 1950s and continued to do so over the years until his latest work about the von Neumann–Heisenberg Question. I will survey various aspects of the theory.

Stefaan Vaes: *Ergodicity and type of nonsingular Bernoulli actions*

I present a joint work with Michael Björklund and Zemer Kosloff on nonsingular Bernoulli actions. These are the translation actions of a discrete group G on the product space $\{0,1\}^G$ equipped with the product of the probability measures μ_g on $\{0,1\}$. We prove in almost complete generality that such an action is either dissipative or weakly mixing, and we determine its Krieger type. In particular, we prove that the group of integers does not admit a Bernoulli action of type II_∞. We prove that a group G admits a Bernoulli action of type III_1 if and only if G has nonzero first L^2-cohomology. We also prove that type III_λ only arises when G has more than one end.

Alain Valette: *From Kadison-Singer to Ramanujan, after Markus-Spielman-Srivastava*

In their celebrated solution of the Kadison-Singer problem, Markus, Spielman and Srivastava established a result that also solved another famous open question (due to Lubotzky, Phillips and Sarnak): for every $d > 2$, there exist infinite families of d-regular Ramanujan graphs. We will explain how the MSS techniques allow to prove the following case of the Bilu–Linial conjecture: every d-regular, bipartite, Ramanujan graph admits a double cover which is Ramanujan. Iterating produces the desired infinite family — a purely existence proof. The proof is a mixture of algebraic graph theory and elementary probability theory, so that the term "Ramanujan graph" might not be so well-chosen after all.

Stuart White: *Close operator algebras*

In the 1970s, Kadison and Kastler introduced a metric on subalgebras of a fixed operator algebra, and conjectured that sufficiently close algebras should be isomorphic. I'll give on overview of the state of the art of this conjecture, and highlight some open problems.

Wilhelm Winter: *Amenability and approximations of C^*-algebras*

I will try to put into context several well-known and one or two new facts around completely positive approximations of C^*-algebras. I will also indicate how pertinent properties, including K-theory data, can be described in terms of systems of completely positive approximations. This last part is joint work in progress with Kristin Courtney.