We write the value of the optimal self-financing continuous portfolio as $V = \tilde{\theta}_0 X + \tilde{\theta}_1 S$. The discrete portfolio is $V' = \theta_0 X + \theta_1 S$. The functions θ_0, θ_1 are constants except at $t_1, t_2, \ldots$ where the jump.

A possible strategy would be to choose $\theta_1(t_k) = \tilde{\theta}_1(t_k)$ (we are not claiming that it is optimal). That (θ_0, θ_1) is self-financing is equivalent to saying that V' is continuous. Therefore θ_0 must obey (explain)

$$\theta_0(t_k) - \theta_0(t_{k-1}) = \frac{S(t_k)}{X(t_k)} \left(\theta_1(t_k) - \theta_1(t_{k-1})\right).$$

On the other the self-financing property of $(\tilde{\theta}_0, \tilde{\theta}_1)$ implies that

$$V(t_k, S(t_k)) - V(t_{k-1}, S(t_{k-1})) = \int_{t_{k-1}}^{t_k} \tilde{\theta}_0 dX + \tilde{\theta}_1 dS.$$

Use this and the expression for V to show that if we define

$$R_{k-1} = \tilde{\theta}_0(t_{k-1})(X(t_k) - X(t_{k-1})) + \tilde{\theta}_1(t_{k-1})(S(t_k) - S(t_{k-1})) - \int_{t_{k-1}}^{t_k} \tilde{\theta}_0 dX + \tilde{\theta}_1 dS$$

then (assuming $\theta_0(0) = \tilde{\theta}_0(0)$) we have

$$\theta_0(t_k) = \tilde{\theta}_0(t_k) + \sum_{j=1}^{k-1} \frac{R_j}{X(t_{j+1})}.$$

Therefore (explain)

$$V'(t_k, S(t_k)) - V(t_k, S(t_k)) = X(t_k) \sum_{j=1}^{k-1} \frac{R_j}{X(t_{j+1})}.$$

Now argue that $E\left((V - V')^2\right)$ tends to zero as $\Delta t \to 0$.