Facitliste til eksamensopgaverne

Eksamen vinteren 2000-01

1(a). Fx $\ell : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. **1(b).** $x - y + z = 1$. **1(c).** $(1, 1, 1)$.

2(i). $\frac{1}{2}x^4 + \frac{1}{2}\cos(2x)$. **2(ii).** $-\frac{1}{4}(2x+3)^{-2}$. **2(iii).** $\frac{2}{3}(\ln x)^{3/2}$.

3(a). Vi har $y = \ln u$ med $u = x + \sqrt{1+x^2}$.
Her er $du/dx = 1 + (2x)/(2\sqrt{1+x^2}) = (\sqrt{1+x^2} + x)/\sqrt{1+x^2} = u/\sqrt{1+x^2}$.
Derfor er $dy/dx = (dy/du)(du/dx) = (1/u)(u/\sqrt{1+x^2}) = 1/\sqrt{1+x^2}$.
3(b). $\Big[\ln(x + \sqrt{1+x^2})\Big]_{3/4}^{4/3} = \ln(\frac{4}{3} + \frac{5}{3}) - \ln(\frac{3}{4} + \frac{5}{4}) = \ln 3 - \ln 2 = \ln(3/2)$.

4(i). $y = t + 1$. **4(ii).** $\cos v = 5/(\sqrt{5}\sqrt{10}) = 1/\sqrt{2}$, så $v = \pi/4 = 45^\circ$.

5(a). $k = \frac{1}{2}$. **5(b).** $y = \sin t + \frac{1}{2}t \sin t$.

6(a). $\partial f/\partial x = 4x - 6y - 2$, $\partial f/\partial y = 10y - 6x + 2$. **6(b).** $(x_0, y_0) = (2, 1)$.

Fortsættes side 2

Eksamen juni 2000

1(i). Areal$= -\int_0^4 (x^2 - 4x)dx = 32/3$. **1(ii).** Rumfang$= \pi \int_0^4 (x^4 - 8x^3 + 16x^2) = 512\pi/15$.

2(i). $\partial f/\partial x = x^2 + y$, $\partial f/\partial y = y + x$. **2(ii).** $(0,0)$ og $(1,-1)$.
2(iii). $z = 19/6 + 3(x-2) + (y+1)$, eller $3x + y - z = 11/6$.

3(i). Ikke A og C, da $\det(A) = \det(C) = 0$. Men B har: $B^{-1} = \begin{pmatrix} 3/2 & -2 \\ -1/2 & 1 \end{pmatrix}$.
3(ii). $\begin{pmatrix} 4 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 3 & 12 \end{pmatrix} = \begin{pmatrix} 16 & 64 \\ 10 & 40 \end{pmatrix}$. **3(iii).** $\vec{u} = \begin{pmatrix} 3/2 & -2 \\ -1/2 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

4(i). $y = 1 + Ke^{x^2}$. **4(ii).** $y = 1 + (2 - 1/e)e^{x^2}$.

5(i). $k = -2$. **5(ii).** $y = c_1e^{7x} + c_2e^{-5x}$. **5(iii).** $y = c_1e^{7x} + c_2e^{-5x} - 2e^{2x}$.
5(iv). $y = \frac{17}{6}e^{7x} + \frac{13}{6}e^{-5x} - 2e^{2x}$.

6(i). Projektion$(P) = (0,-1,1)$. **6(ii).** $\text{dist}(P,\ell) = 4\sqrt{5}$. **6(iii).** Fx $-3x + 5y + 6z = 1$.

Eksamen januar 2000

1(a). $\frac{3}{8}x^8 + \frac{4}{3}x^3 - 5x + K$. **1(b).** $-\frac{1}{2}\cos(x^2 + 4x) + K$.

2(i). $\begin{pmatrix} 12 \\ 27 \end{pmatrix}$. **2(ii).** Projektion$(\vec{b}) = \begin{pmatrix} 4/5 \\ -2/5 \end{pmatrix}$. **2(iii).** $x = 5$, $y = -2$. **2(iv).** $t = 40/21$.

3(ii). $\int_0^4 \sqrt{x}\,dx - \int_2^4 \sqrt{x-2}\,dx = \frac{2}{3}4^{3/2} - \frac{2}{3}2^{3/2} = \frac{4}{3}(4 - \sqrt{2})$.
3(iii). $\pi \int_0^4 (\sqrt{x})^2 dx - \pi \int_2^4 (\sqrt{x-2})^2 dx = 6\pi$.

4(i). $a^2 + a + 1 \neq 0$, dvs $a \neq (-1 \pm \sqrt{5})/2$. **4(ii).** $f_2^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

5(i). $y = (\frac{1}{6}x^6 + \frac{1}{4}x^4 + K)/(x^2 + 1)$. **5(ii).** $y = (\frac{1}{6}x^6 + \frac{1}{4}x^4 + 7)/(x^2 + 1)$.

6(i). α og β er ikke parallelle, da normalerne ikke er proportionale. Skæringslinien: Fx $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} -5 \\ 13 \\ -1 \end{pmatrix}$. **6(ii).** $\cos(\text{vinklen}) = 15/(\sqrt{14}\sqrt{30}) = \sqrt{105}/14$.
6(iii). Projektion$(P) = (-1/2, 8/5, 33/10)$.

7(i). x-aksen: $t = 1$, y-aksen: $t = -1$ og $t = -10$. **7(ii).** vandret: $t = 0$, lodret: $t = -11/2$.
7(iii). $t = 0$ og $t = -11$.

Eksamen maj 1999

1. $\vec{b} = \binom{24/5}{-28/5}$.

2(i). $x = 2$. **2(ii).** Areal$(M) = -\int_0^2 (x^3 - 8)dx = 12$.
2(iii). Rumfang$= \pi \int_0^2 (x^3 - 8)^2\, dx = 512\pi/7$.

3(i). $y = \pm\sqrt{\ln(x^2 + 1) + k}$. **3(ii).** $y = \sqrt{\ln(x^2 + 1) + 4}$.

4(i). cos(vinklen) $= -1/\sqrt{30}$. **4(ii).** $(2, -1, 1)$. **4(iii).** $4x + 2y + 3z = 9$.
4(iv). $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 9/2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$.

5(i). $\partial f/\partial x = 2x - 2y, \partial f/\partial y = -2x + 3y^2$. **5(ii).** $(x, y) = (0, 0)$ og $(x, y) = (2/3, 2/3)$.
5(iii). Maximum $= 1$, minimum $= -4/27$.

6(i). $(0, -2)$ med $\vec{v} = \binom{-4}{5}$ (for $t = -2$); $(-2, 0)$ med $\vec{v} = \binom{-2\sqrt{2}}{2}$ (for $t = -\sqrt{2}$);
$(-4, 0)$ med $\vec{v} = \binom{0}{-1}$ (for $t = 0$); $(-2, 0)$ med $\vec{v} = \binom{2\sqrt{2}}{2}$ (for $t = \sqrt{2}$);
$(0, 2)$ med $\vec{v} = \binom{4}{5}$ (for $t = 2$). **6(ii).** $t = \pm\sqrt{\frac{2}{3}}$ (vandret) og $t = 0$ (lodret).

Eksamen januar 1999

1(i). $x^3/3 + \frac{4}{3}x^{3/2} - 3x(\ln x - 1) + C$. **1(ii).** $7 \ln |x - 5| + 3 \ln |x + 2| + C$.
1(iii). $2\big(\sin(2) - \sin(1)\big)$.

2(i). cos(vinklen) $= -1/\sqrt{30}$. **2(ii).** $(2, -1, 1)$. **2(iii).** $4x + 2y + 3z = 9$.
2(iv). $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 9/2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$.

3(i). $c_1e^x + c_2e^{-4x}$. **3(ii).** $c_1e^x + c_2e^{-4x} + 2x^2 - 3$. **3(iii).** $-\frac{4}{5}e^x + \frac{4}{5}e^{-4x} + 2x^2 - 3$.

4(i). $f\binom{2}{3} = \binom{-7}{14}$. **4(ii).** $\vec{u} = \binom{5}{-2}$. **4(iii).** $f^{-1}\binom{x}{y} = \begin{pmatrix} -4/7 & -1/7 \\ 1/7 & 2/7 \end{pmatrix} \binom{x}{y}$.

5(i). $\partial f/\partial x = 2x - y + 1, \partial f/\partial y = -x + 2y - 12$. **5(ii).** $(x, y) = (10/3, 23/3)$.
5(iii). $z - 5 = 8(x - 4) - 14(y - 1)$, eller $8x - 14y - z = 13$.

6(i). $5t^2 - 21t + 18 = 0$, altså $t = 3$ og $t = 6/5$.
6(ii). $t = 7/3$. **6(iii).** $t = 11/3$ og $t = 1$.