

18.022: Multivariable calculus - problem set 4 - fall 2006

Due by 1:45 PM, Room 2-106, Friday 10/6.

Note that we use 3rd edition of the text for reference and earlier editions may number problems differently. Whilst you may attempt problems in any order, graders will appreciate if you hand in your problems in order. By all means, please try to help the graders.

1. (10 points) This problem shows that the opposite of Thm. 3.10 is not true. We consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = |xy|$. Show that
 - (a) f is differentiable at $(0, 0)$.
 - (b) the partial derivatives $\partial f / \partial x$ and $\partial f / \partial y$ are *not* continuous in any neighborhood of $(0, 0)$.

2. (10 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin x$, if x is rational, and $f(x) = 0$, otherwise. Determine at what points $a \in \mathbb{R}$ the function f is continuous.

3. (10 points) Find a function $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = 3y^2 - 2x \cos y \quad \text{and} \quad \frac{\partial f}{\partial y} = 6xy + x^2 \sin y + 2.$$

4. (5 points) 2.2.3
5. (5 points) 2.2.4
6. (5 points) 2.2.31
7. (5 points) 2.2.33
8. (5 points) 2.2.40
9. (5 points) 2.2.46
10. (5 points) 2.3.21
11. (5 points) 2.3.23
12. (5 points) 2.3.24
13. (5 points) 2.3.25
14. (5 points) 2.3.27
15. (5 points) 2.3.28
16. (5 points) 2.3.30
17. (5 points) 2.3.32

Total: 100 points