18.022: Multivariable calculus - problem set 5 - fall 2006

Due by 1:45 PM, Room 2-106, Friday 10/13.

Note that we use 3rd edition of the text for reference and earlier editions may number problems differently. Whilst you may attempt problems in any order, graders will appreciate if you hand in your problems in order. By all means, please try to help the graders.

1. (10 points) Find a recursive formula for a sequence of points (x_0, y_0), $(x_1, y_1), \ldots, (x_k, y_k), \ldots$ whose limit, if it exists, is a point of intersection of the two curves:

$$x^2 + y^2 = 1 \quad \text{(unit circle)}$$

$$x^2(x+1) = y^2 \quad \text{(nodal curve).}$$

2. (10 points) Let $F : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function $F(x, y) = \sin(xy) + x^3 + y^3$. Find a recursive formula for a sequence of points $(x_0, y_0), (x_1, y_1), \ldots,$ $(x_k, y_k), \ldots$ whose limit, if it exists, is a *critical point* for the function F, that is, a point (x, y) such that $DF(x, y) = (0, 0)$.

3. (20 points) Let $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be of class $\mathcal{C}^1$ and assume that $F(3, -1, 2) = (0, 0)$ and that

$$DF(3, -1, 2) = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

   (a) Show that there is a neighborhood U of 3 in $\mathbb{R}$ and a function $f : U \longrightarrow \mathbb{R}^2$ of class $\mathcal{C}^1$ such that $f(3) = (-1, 2)$ and such that, for all x in U, $F(x, f(x)) = (0, 0)$.

   (b) Find $Df(3)$.

4. (5 points) 2.4.20

5. (5 points) 2.4.24(b)

6. (5 points) 2.4.25(b)

7. (5 points) 2.5.11

8. (5 points) 2.5.20

9. (5 points) 2.5.21

10. (5 points) 2.5.23

11. (5 points) 2.5.28
12. (5 points) 2.5.35
13. (5 points) 2.6.23
14. (5 points) 2.6.24
15. (5 points) 2.6.34

Total: 100 points