# 18.022 Problem set 5 (v.2)

1. Define
$$\begin{cases} f_1(x,y) = x^2 + y^2 - 1, \\ f_2(x,y) = x^2(x+1) - y^2. \end{cases}$$
We have that
$$\begin{aligned} D\mathbf{f}(x,y) &= \begin{pmatrix} 2x & 2y \\ 3x^2+2x & -2y \end{pmatrix} \\ \iff (D\mathbf{f}(x,y))^{-1} &= \frac{1}{8xy+6x^2y} \cdot \begin{pmatrix} 2y & 2y \\ 3x^2+2x & -2x \end{pmatrix}. \end{aligned}$$
To find points $(x_0,y_0),(x_1,y_1),\ldots$ with desired properties, we use Newton's method:
$$\begin{aligned} &\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} - (D\mathbf{f}(x_i,y_i))^{-1} \cdot \begin{pmatrix} f_1(x_i,y_i) \\ f_2(x_i,y_i) \end{pmatrix} \\ &= \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \frac{1}{8x_iy_i+6x_i^2y_i} \cdot \begin{pmatrix} 2y_i & 2y_i \\ 3x_i^2+2x_i & -2x_i \end{pmatrix} \cdot \begin{pmatrix} x_i^2+y_i^2-1 \\ x_i^3+x_i^2-y_i^2 \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{8x_i^2y_i+6x_i^3y_i-2y_i(x_i^2+y_i^2-1+x_i^3+x_i^2-y_i^2)}{8x_iy_i+6x_i^2y_i} \\ \\ \dfrac{8x_iy_i^2+6x_i^2y_i^2-(3x_i^2+2x_i)(x_i^2+y_i^2-1)+2x_i(x_i^3+x_i^2-y_i^2)}{8x_iy_i+6x_i^2y_i} \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{2x_i^2+2x_i^3+1}{4x_i+3x_i^2} \\ \\ \dfrac{4y_i^2+3x_iy_i^2-x_i^3+3x_i+2}{8y_i+6x_iy_i} \end{pmatrix} \end{aligned}$$
To conclude,
$$\begin{cases} x_{i+1} = \dfrac{2x_i^2+2x_i^3+1}{4x_i+3x_i^2}, \\ \\ y_{i+1} = \dfrac{4y_i^2+3x_iy_i^2-x_i^3+3x_i+2}{8y_i+6x_iy_i}. \end{cases}$$
*Remark.* Setting $x_0 = y_0 = 0.5$, this yields the approximation
$$(x,y) \approx (0.61803568, 0.78615138).$$

2. We have that

$$\begin{aligned} & F(x,y) = \sin(xy) + x^3 + y^3 \\ \Longrightarrow\ & \nabla F(x,y) = (y\cos(xy) + 3x^2, x\cos(xy) + 3y^2). \end{aligned}$$

We obtain that

$$\begin{aligned} D(\nabla F(x,y)) &= \begin{pmatrix} -y^2\sin(xy) + 6x & \cos(xy) - xy\sin(xy) \\ \cos(xy) - xy\sin(xy) & -x^2\sin(xy) + 6y \end{pmatrix} \\ \Longrightarrow\ (D\mathbf{f}(x,y))^{-1} &= \frac{1}{\det D(\nabla F(x,y))} \\ &\cdot \begin{pmatrix} -x^2\sin(xy) + 6y & -\cos(xy) + xy\sin(xy) \\ -\cos(xy) + xy\sin(xy) & -y^2\sin(xy) + 6x \end{pmatrix}. \end{aligned}$$

Note that

$$\begin{aligned} & \det D(\nabla F(x,y)) \\ = \ & (-y^2\sin(xy) + 6x)(-x^2\sin(xy) + 6y) - (\cos(xy) - xy\sin(xy))^2 \\ = \ & 2(xy\cos(xy) - 3x^3 - 3y^3)\sin(xy) + 36xy - \cos^2(xy). \end{aligned}$$

To find points $(x_0, y_0), (x_1, y_1), \ldots$ with desired properties, we use Newton's method:

$$\begin{aligned} & \begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} - (D(\nabla f(x_i, y_i)))^{-1} \cdot \begin{pmatrix} f_x(x_i, y_i) \\ f_y(x_i, y_i) \end{pmatrix} \\ = & \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \frac{1}{\det D(\nabla F(x_i, y_i))} \\ & \cdot \begin{pmatrix} -x_i^2\sin(x_iy_i) + 6y_i & -\cos(x_iy_i) + x_iy_i\sin(x_iy_i) \\ -\cos(x_iy_i) + x_iy_i\sin(x_iy_i) & -y_i^2\sin(x_iy_i) + 6x_i \end{pmatrix} \\ & \cdot \begin{pmatrix} y_i\cos(x_iy_i) + 3x_i^2 \\ x_i\cos(x_iy_i) + 3y_i^2 \end{pmatrix} \\ = & \frac{1}{2(x_iy_i\cos(x_iy_i) - 3x_i^3 - 3y_i^3)\sin(x_iy_i) + 36x_iy_i - \cos^2(x_iy_i)} \\ & \cdot \begin{pmatrix} (2x_i^2y_i\cos(x_iy_i) - 9x_iy_i^3 - 3x_i^4)\sin(x_iy_i) + 18x_i^2y_i - 3y_i^2\cos(x_iy_i) \\ (2x_iy_i^2\cos(x_iy_i) - 9x_i^3y_i - 3y_i^4)\sin(x_iy_i) + 18x_iy_i^2 - 3x_i^2\cos(x_iy_i) \end{pmatrix}. \end{aligned}$$

*Remark.* Setting $x_0 = y_0 = -1$, this yields the approximation $x = y \approx -0.33132683$.

3. (a) By the Implicit Function Theorem, it suffices to prove that

$$\begin{vmatrix} \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{vmatrix} \neq 0$$

at the point $(x, y, z) = (3, -1, 2)$. By assumption,

$$\begin{vmatrix} \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3,$$

which is indeed nonzero.

(b) Write $f(x) = (f_1(x), f_2(x))$. To compute $Df(3)$, we use the chain rule:

$$\begin{aligned} 0 &= \frac{d}{dx} F_1(x, f_1(x), f_2(x)) = \frac{\partial F_1}{\partial x} + \frac{\partial F_1}{\partial y} \cdot \frac{df_1}{dx} + \frac{\partial F_1}{\partial z} \cdot \frac{df_2}{dx} \\ &= 1 + 2f_1'(x) + f_2'(x), \\ 0 &= \frac{d}{dx} F_2(x, f_1(x), f_2(x)) = \frac{\partial F_2}{\partial x} + \frac{\partial F_2}{\partial y} \cdot \frac{df_1}{dx} + \frac{\partial F_2}{\partial z} \cdot \frac{df_2}{dx} \\ &= 1 - f_1'(x) + f_2'(x). \end{aligned}$$

We conclude that

$$\begin{aligned} 0 &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} f_1'(3) \\ f_2'(3) \end{pmatrix} \\ \Longleftrightarrow \begin{pmatrix} f_1'(3) \\ f_2'(3) \end{pmatrix} &= -\frac{1}{3} \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \end{aligned}$$

4. (Exercise 2.4.20)

   (a) If $f(x, y, z) = ax^2 + by^2 + cz^2$, then $f_{xx} = 2a$, $f_{yy} = 2b$, and $f_{zz} = 2c$; hence

   $$f_{xx} + f_{yy} + f_{zz} = 2a + 2b + 2c = 2(a + b + c).$$

   As a consequence, $f$ is harmonic if and only if $a+b+c = 0$. In particular, $f(x, y, z) = x^2 + y^2 - 2z^2$ is harmonic, whereas $f(x, y, z) = x^2 - y^2 + z^2$ is not.

   (b) If $f_{x_1, x_2, \ldots, x_n} = a_1 x_1^2 + a_2 x_2^2 + \cdots + a_n x_n^2$, then $f_{x_i x_i} = 2a_i$ and hence

   $$f_{x_1 x_1} + f_{x_2 x_2} + \cdots + f_{x_n x_n} = 2(a_1 + a_2 + \cdots a_n).$$

   It follows that $f$ is harmonic whenever $a_1 + a_2 + \cdots + a_n = 0$. For example, $f(x_1, x_2, \ldots, x_n) = (n-1)x_n^2 - x_1^2 - x_2^2 - \cdots - x_{n-1}^2$ is harmonic.

5. (Exercise 2.4.24b)

   We consider the surface $e^z \cos y = \cos x$. Assuming $\cos x \neq 0$ and $\cos y \neq 0$, we may rewrite this as

   $$z = \ln \cos x - \ln \cos y.$$

Differentiating, we obtain that

$$z_x = -\frac{\sin x}{\cos x}; z_y = \frac{\sin y}{\cos y}; z_{xx} = -\frac{1}{\cos^2 x}; z_{xy} = 0; z_{yy} = \frac{1}{\cos^2 y}.$$

As a consequence,

$$\begin{aligned} & (1+z_y^2)z_{xx} + (1+z_x^2)z_{yy} - 2z_x z_y z_{xy} \\ = & -\left(1+\frac{\sin^2 y}{\cos^2 y}\right)\frac{1}{\cos^2 x} + \left(1+\frac{\sin^2 x}{\cos^2 x}\right)\frac{1}{\cos^2 y} - 0 \\ = & -\frac{1}{\cos^2 y \cos^2 x} + \frac{1}{\cos^2 x \cos^2 y} = 0. \end{aligned}$$

It follows that this surface is minimal.

6. (Exercise 2.4.25b)

We consider the surface $f(x,y,z) = x = y\tan z$. Differentiating, we obtain that

$$x_y = \tan z; x_z = \frac{y}{\cos^2 z}; x_{yy} = 0; x_{yz} = \frac{1}{\cos^2 z}; x_{zz} = \frac{2y\sin z}{\cos^3 z}.$$

As a consequence,

$$\begin{aligned} & (1+x_z^2)x_{yy} + (1+x_y^2)x_{zz} - 2x_y x_z x_{yz} \\ = & \left(1+\frac{y^2}{\cos^4 z}\right)\cdot 0 + \left(1+\frac{\sin^2 z}{\cos^2 z}\right)\cdot\frac{2y\sin z}{\cos^3 z} - 2\cdot\frac{\sin z}{\cos z}\cdot\frac{y}{\cos^2 z}\cdot\frac{1}{\cos^2 z} \\ = & \frac{2y\sin z}{\cos^5 z} - \frac{2y\sin z}{\cos^5 z} = 0. \end{aligned}$$

It follows that this surface is minimal.

7. (Exercise 2.5.11)

Let $u = \frac{xy}{x^2+y^2}$ and $w = f(u) = f\left(\frac{xy}{x^2+y^2}\right)$. We have that

$$\begin{aligned} \frac{\partial w}{\partial x} &= \frac{df}{du}\frac{\partial u}{\partial x} = \frac{df}{du}\left(\frac{x^2y+y^3-2x^2y}{(x^2+y^2)^2}\right) = \frac{df}{du}\left(\frac{y(y^2-x^2)}{(x^2+y^2)^2}\right), \\ \frac{\partial w}{\partial y} &= \frac{df}{du}\frac{\partial u}{\partial y} = \frac{df}{du}\left(\frac{x^3+xy^2-2xy^2}{(x^2+y^2)^2}\right) = \frac{df}{du}\left(\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right). \end{aligned}$$

Thus

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = \frac{df}{du}\left(\frac{xy(y^2-x^2)}{(x^2+y^2)^2}\right) + \frac{df}{du}\left(\frac{xy(x^2-y^2)}{(x^2+y^2)^2}\right) = 0.$$

8. (Exercise 2.5.20)

If $\mathbf{g} : \mathbb{R}^3 \to \mathbb{R}^2$ and $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ are differentiable functions, then

$$D(\mathbf{f} \circ \mathbf{g})(x, y, z) = D\mathbf{f}(\mathbf{g}(x, y, z)) \cdot D\mathbf{g}(x, y, z).$$

In this particular case, $(x, y, z) = (1, -1, 3)$, $\mathbf{g}(1, -1, 3) = (2, 5)$, and

$$D\mathbf{g}(1, -1, 3) = \begin{pmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{pmatrix}.$$

Moreover, $\mathbf{f}(x, y) = (2xy, 3x - y + 5)$, which yields that

$$D\mathbf{f}(x, y) = \begin{pmatrix} 2y & 2x \\ 3 & -1 \end{pmatrix} \Longrightarrow D\mathbf{f}(2, 5) = \begin{pmatrix} 10 & 4 \\ 3 & -1 \end{pmatrix}.$$

Thus

$$D(\mathbf{f} \circ \mathbf{g})(1, -1, 3) = \begin{pmatrix} 10 & 4 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 26 & -10 & 28 \\ -1 & -3 & -7 \end{pmatrix}.$$

9. (Exercise 2.5.21)

(a)

$$\begin{aligned} D(\mathbf{f} \circ \mathbf{g})(1, 2) &= D\mathbf{f}(\mathbf{g}(1, 2)) \cdot D\mathbf{g}(1, 2) \\ &= D\mathbf{f}(3, 5) \cdot D\mathbf{g}(1, 2) \\ &= \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 31 & 44 \end{pmatrix}. \end{aligned}$$

(b)

$$\begin{aligned} D(\mathbf{g} \circ \mathbf{f})(4, 1) &= D\mathbf{g}(\mathbf{f}(4, 1)) \cdot D\mathbf{f}(4, 1) \\ &= D\mathbf{g}(1, 2) \cdot D\mathbf{f}(4, 1) \\ &= \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 13 \\ 2 & 31 \end{pmatrix}. \end{aligned}$$

10. (Exercise 2.5.23)

(a) We have that

$$\begin{aligned}
\frac{\partial^2}{\partial x^2} &= \left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\\
&= \cos\theta\frac{\partial}{\partial r}\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\\
&\quad -\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\\
&= \cos^2\theta\frac{\partial^2}{\partial r^2} - \cos\theta\sin\theta\left(-\frac{1}{r^2}\frac{\partial}{\partial\theta} + \frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}\right)\\
&\quad -\frac{\sin\theta}{r}\left(-\sin\theta\frac{\partial}{\partial r} + \cos\theta\frac{\partial^2}{\partial\theta\partial r}\right)\\
&\quad +\frac{\sin\theta}{r}\left(\frac{\cos\theta}{r}\frac{\partial}{\partial\theta} + \frac{\sin\theta}{r}\frac{\partial^2}{\partial\theta^2}\right)\\
&= \cos^2\theta\frac{\partial^2}{\partial r^2} + \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta} - \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}\\
&\quad +\frac{\sin^2\theta}{r}\frac{\partial}{\partial r} + \frac{\sin^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}.
\end{aligned}$$

We may compute $\partial^2/\partial y^2$ in the same manner, but we may also observe that we obtain $\partial/\partial y$ from $\partial/\partial x$ by replacing $\theta$ with $\theta - \pi/2$; namely, $\cos(\theta-\pi/2) = \sin\theta$ and $\sin(\theta-\pi/2) = -\cos\theta$. Thus we obtain $\partial^2/\partial y^2$ from $\partial^2/\partial x^2$ via the same modification:

$$\begin{aligned}
\frac{\partial^2}{\partial y^2} &= \sin^2\theta\frac{\partial^2}{\partial r^2} - \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta} + \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}\\
&\quad +\frac{\cos^2\theta}{r}\frac{\partial}{\partial r} + \frac{\cos^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}.
\end{aligned}$$

(b) We have that

$$\begin{aligned}
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} &= \cos^2\theta\frac{\partial^2}{\partial r^2} + \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta} - \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}\\
&\quad +\frac{\sin^2\theta}{r}\frac{\partial}{\partial r} + \frac{\sin^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}\\
&\quad +\sin^2\theta\frac{\partial^2}{\partial r^2} - \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial}{\partial\theta} + \frac{2\cos\theta\sin\theta}{r}\frac{\partial^2}{\partial\theta\partial r}\\
&\quad +\frac{\cos^2\theta}{r}\frac{\partial}{\partial r} + \frac{\cos^2\theta}{r^2}\frac{\partial^2}{\partial\theta^2}\\
&= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.
\end{aligned}$$

11. (Exercise 2.5.28)

(a) Since $F(x,y,z(x,y)) = 0$, we have that

$$\begin{aligned} 0 &= \frac{\partial}{\partial x}(F(x,y,z(x,y))) = F_x + \frac{\partial z}{\partial x}F_z, \\ 0 &= \frac{\partial}{\partial y}(F(x,y,z(x,y))) = F_y + \frac{\partial z}{\partial y}F_z. \end{aligned}$$

As a consequence, $\partial z/\partial x = -F_x/F_z$ and $\partial z/\partial y = -F_y/F_z$.

(b) Define $F(x,y,z) = xyz - 2$. We have that $F_x = yz$, $F_y = xz$, and $F_z = xy$. With $F(x,y,z) = 0$, part (a) yields that

$$\begin{aligned} \frac{\partial z}{\partial x} &= -\frac{F_x}{F_z} = -\frac{yz}{xy} = -\frac{xyz}{x^2y} = -\frac{2}{x^2y}, \\ \frac{\partial z}{\partial y} &= -\frac{F_y}{F_z} = -\frac{xz}{xy} = -\frac{xyz}{xy^2} = -\frac{2}{xy^2}. \end{aligned}$$

Solving for $z$ directly, we obtain that $z = 2/xy$. Differentiation yields that $z_x = -2/x^2y$ and $z_y = -2/xy^2$.

12. (Exercise 2.5.35)

By the previous exercise, we have that

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = \frac{-F_y}{F_x}\cdot\frac{-F_z}{F_y}\cdot\frac{-F_x}{F_z} = -1.$$

13. (Exercise 2.6.23)

Let $f(x,y,z) = 9x^2 - 45y^2 + 5z^2 - 45$. Then $\nabla f = (18x, -90y, 10z)$. The tangent plane at the point $(x,y,z)$ is parallel to the plane $x+5y-2z = 7$ if and only if $\nabla f$ is a multiple of $(1,5,-2)$. Equivalently,

$$\frac{18x}{1} = \frac{-90y}{5} = \frac{10z}{-2} \iff 18x = -18y = -5z;$$

thus $(x,y,z) = (5\lambda, -5\lambda, -18\lambda)$ for some $\lambda \in \mathbb{R}$. Now,

$$\begin{aligned} 0 &= f(5\lambda, -5\lambda, -18\lambda) = 9(5\lambda)^2 - 45(-5\lambda)^2 + 5(-18\lambda)^2 - 45 \\ &= \lambda^2 \cdot 45(5 - 25 + 36) - 45 = 45(16\lambda^2 - 1). \end{aligned}$$

As a consequence, $\lambda = \pm 1/4$, which implies that

$$(x,y,z) = \pm(5/4, -5/4, -9/2).$$

14. (Exercise 2.6.24)

Write $f(x,y,z) = z - 7x^2 + 12x + 5y^2$ and $g(x,y,z) = xyz^2 - 2$. It is clear that $f(2,1,-1) = g(2,1,-1) = 0$. We have that

$$\begin{aligned} \nabla f(x,y,z) &= (-14x+12, 10y, 1) \Longrightarrow \nabla f(2,1,-1) = (-16,10,1) \\ \nabla g(x,y,z) &= (yz^2, xz^2, 2xyz) \Longrightarrow \nabla g(2,1,-1) = (1,2,-4). \end{aligned}$$

Since

$$\begin{aligned} \nabla f(2,1,-1) \cdot \nabla g(2,1,-1) &= (-16,10,1) \cdot (1,2,-4) \\ &= -16 + 20 - 4 = 0, \end{aligned}$$

it follows that the tangent planes to $f(x,y,z) = 0$ and $g(x,y,z) = 0$ intersect orthogonally at $(2,1,-1)$.

15. (Exercise 2.6.34)

(a) Let $f(x,y,z) = x^3z + x^2y^2 + \sin(yz) + 3$. To compute the plane tangent to the surface $f(x,y,z) = 0$ at the point $(-1,0,3)$, we first note that $f(-1,0,3) = -3 + 0 + 0 + 3 = 0$ and that

$$\begin{aligned} \nabla f(x,y,z) &= (3x^2z + 2xy^2, 2x^2y + z\cos(yz), x^3 + y\cos(yz)) \\ \Longrightarrow \nabla f(-1,0,3) &= (9,3,-1). \end{aligned}$$

As a consequence, the plane is given by

$$9(x+1) + 3(y-0) - 1(z-3) = 0 \Longleftrightarrow 9x + 3y - z = -12.$$

(b) The normal line to the surface at $(-1,0,3)$ is given by $(x,y,z) = (-1,0,3) + t(9,3,-1)$. Equivalently,

$$\begin{cases} x = -1 + 9t, \\ y = 3t, \\ z = 3 - t. \end{cases}$$