18.022: Multivariable calculus - problem set 7(v2) - fall 2006

Due by 1:45 PM, Room 2-106, Friday 10/27.

Note that we use 3rd edition of the text for reference and earlier editions may number problems differently. Whilst you may attempt problems in any order, graders will appreciate if you hand in your problems in order. By all means, please try to help the graders.

1. (10 points) This problem proves the uniqueness statement of Thm. 2.5 on page 201 of the text. Let $\mathbf{r}\colon I \to \mathbb{R}^3$ and $\bar{\mathbf{r}}\colon I \to \mathbb{R}^3$ be two curves of class C^3 parametrized by arclength. Assume that $\kappa(s)$ and $\bar{\kappa}(s)$ are positive and equal, for all $s \in I$, and that $\tau(s)$ and $\bar{\tau}(s)$ are equal, for all $s \in I$. Assume further that there exists $a \in I$ such that $\mathbf{r}(a) = \bar{\mathbf{r}}(a)$, $\mathbf{T}(a) = \bar{\mathbf{T}}(a)$, $\mathbf{N}(a) = \bar{\mathbf{N}}(a)$, and $\mathbf{B}(a) = \bar{\mathbf{B}}(a)$. Show that $\mathbf{r}(s) = \bar{\mathbf{r}}(s)$ for all $s \in I$.

   (Hint: Use the Frenet formulas to show that

   $$||\mathbf{T}(s) - \bar{\mathbf{T}}(s)||^2 + ||\mathbf{N}(s) - \bar{\mathbf{N}}(s)||^2 + ||\mathbf{B}(s) - \bar{\mathbf{B}}(s)||^2$$

   is a constant function of $s \in I$.)

2. (10 points) Let $\mathbf{r}\colon I \to \mathbb{R}^3$ be the helix defined by

   $$\mathbf{r}(s) = a\cos(\frac{s}{c})\mathbf{i} + a\sin(\frac{s}{c})\mathbf{j} + \frac{bs}{c}\mathbf{k},$$

   where $a^2 + b^2 = c^2$.

   (a) Show that $\mathbf{r}$ is parametrized by arclength.

   (b) Find $\mathbf{T}(s)$, $\mathbf{N}(s)$, and $\mathbf{B}(s)$.

   (c) Find $\kappa(s)$ and $\tau(s)$.

3. (5 points) 3.2.16

4. (10 points) 3.2.40

5. (10 points) 3.2.41

6. (5 points) 3.3.3

7. (5 points) 3.3.19

8. (5 points) 3.3.20

9. (5 points) 3.3.23

10. (5 points) 3.3.25

11. (5 points) 3.4.1
12. (5 points) 3.4.3
13. (5 points) 3.4.6
14. (5 points) 3.4.13
15. (5 points) 3.6.43
16. (5 points) 3.6.44

Total: 100 points