18.022: Multivariable calculus - problem set 13 - fall 2006

Due by 1:45 PM, Room 2-106, Friday 12/8.

Note that we use 3rd edition of the text for reference and earlier editions may number problems differently. Whilst you may attempt problems in any order, graders will appreciate if you hand in your problems in order. By all means, please try to help the graders.

1. (25 points) Let C be a smooth curve, and let $\mathbf{x}\colon [a,b] \to \mathbb{R}^3$ be a parametrization of C by arclength. We assume that the curvature $\kappa(s)$ is everywhere non-zero. Let r be a positive number that is everywhere smaller than the radius of curvature $r(s) = 1/\kappa(s)$. The *tube* of radius r around $\mathbf{x}$ is defined to be the image of the map

   $$\mathbf{g}\colon [a,b] \times [0, 2\pi] \to \mathbb{R}^3$$

   given by

   $$\mathbf{g}(s,t) = \mathbf{x}(s) + r\cos t\mathbf{N}(s) + r\sin t\mathbf{B}(s)$$

   where $\mathbf{N}(s)$ is the normal vector of $\mathbf{x}$ and $\mathbf{B}(s)$ is the bi-normal vector of $\mathbf{x}$. We assume that the tube is a smooth surface S.

   (a) Show that

   $$\frac{\partial \mathbf{g}}{\partial s} \times \frac{\partial \mathbf{g}}{\partial t} = r\cos t(r\kappa(s)\cos t - 1)\mathbf{N}(s) + r\sin t(r\kappa(s)\cos t - 1)\mathbf{B}(s).$$

   (b) Show that the area of S is equal to $2\pi r$ times the length of C.

2. (5 points) 6.3.3.
3. (5 points) 6.3.14
4. (5 points) 6.3.15
5. (5 points) 6.3.18
6. (5 points) 7.1.3
7. (5 points) 7.1.8
8. (5 points) 7.1.18
9. (10 points) 7.1.22
10. (10 points) 7.2.3
11. (10 points) 7.2.10
12. (10 points) 7.2.14

Total: 100 points